Rozklad na parciální zlomky + integrace

$$\int \frac{4x^2+6}{x^2-1}\,dx = \int\left(4+\frac{10}{x^2-1}\right)dx = \int\left(4+\frac{5}{x-1}-\frac{5}{x+1}\right)dx = 4x+5\ln|x-1|-5\ln|x+1|+c$$

$0x+10$

$(4x^2+6):(x^2-1)=4$

$-\dfrac{(4x^2-4)}{10}$

ZBYTEK

$$\frac{4x^2+6}{x^2-1} = 4+\frac{10}{x^2-1}$$

$$\frac{10}{x^2-1} = \frac{10}{(x-1)(x+1)} = \frac{A}{(x-1)}+\frac{B}{(x+1)} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)} = \frac{\overbrace{(A+B)}^{=0}x+\overbrace{A-B}^{=10}}{(x-1)(x+1)} = \frac{5}{x-1}-\frac{5}{x+1}$$

$x\neq\{-1,1\}$

$\cdot x^1: A+B=0$

$\cdot x^0: A-B=10$

$2A=10 \rightarrow A=5$

$B=-5$

$$\int \frac{6x-12}{x^2-9}\,dx = \int \frac{6x-12}{(x-3)(x+3)}\,dx = \int \left(\frac{1}{x-3} + \frac{5}{x+3}\right)dx = \ln|x-3| + 5\ln|x+3| + c$$

$x \neq \{-3, 3\}$

$$\frac{6x-12}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3} = \frac{A(x+3)+B(x-3)}{(x-3)(x+3)} = \frac{Ax+Bx+3A-3B}{(x-3)(x+3)} =$$

$$= \frac{1}{x-3} + \frac{5}{x+3}$$

mx: $6 = A + B \quad /\cdot 3$

mx^0: $-12 = 3A - 3B$

$6 = 6A$

$A = 1$

$B = 5$

$$\int \frac{x^4+3x^3+2x-7}{x^2+1}\,dx = \int \left(x^2+3x-1+\frac{-x-6}{x^2+1}\right)dx = \frac{x^3}{3}+3\frac{x^2}{2}-x+(*)$$

$$(x^4+3x^3+2x-7):(x^2+1)=x^2+3x-1$$

$$-(x^4+x^2)$$

$$3x^3-x^2+2x-7$$

$$-(3x^3+3x)$$

$$-x^2-x-7$$

$$-(-x^2-1)$$

$$-x-6$$

$$(*)\quad \int \frac{-x-6}{x^2+1}\,dx = -\frac{1}{2}\int \frac{2x}{x^2+1}\,dx - 6\int \frac{1}{x^2+1}\,dx =$$

$$= -\frac{1}{2}\ln(x^2+1) - 6\operatorname{arctg} x + c$$

$$t = x^2+1$$

$$dt = 2x\,dx$$

$$\int \frac{32x^2+32}{(x-1)^2(x-9)}\,dx = \int\left(\frac{-8}{(x-1)^2} - \frac{9}{x-1} + \frac{41}{x-9}\right)dx = -8\frac{(x-1)^{-1}}{-1} - 9\ln|x-1| + 41\ln|x-9| + c$$

$x \neq 1; 9$

$$\frac{32x^2+32}{(x-1)^2(x-9)} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x-9)} = \frac{A(x-9)+B(x-1)(x-9)+C(x-1)^2}{(x-1)^2(x-9)}$$

čitatel: $32x^2+32 = A(x-9) + B(x^2-10x+9) + C(x^2-2x+1)$

x^2: $32 = B + C$

x: $0 = A - 10B - 2C \;/\cdot 9$

x: $32 = -9A + 9B + C$

$32 = B + C \;/\cdot 17$

$32 = -81B - 17C$

$576 = -64B \rightarrow B = -9$

$C = 41$

$A = 10B + 2C = -90 + 82 = -8$

$$\frac{2x^2-2x+5}{(x-1)^2(x^2+4)} = \frac{A}{(x-1)^2} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4} = \frac{1}{(x-1)^2} + \frac{1}{x^2+4}$$

$$\frac{A(x^2+4) + B(x-1)(x^2+4) + (Cx+D)(x^2-2x+1)}{(x-1)^2(x^2+4)} \quad (x-1)^2$$

citatelé: $2x^2-2x+5 = A(x^2+4) + B(x^3-x^2+4x-4) + Cx^3-2Cx^2+Cx+Dx^2-2Dx+D$

- x^3: $0 = B+C$
- x^2: $2 = A-B-2C+D$
- x: $-2 = 4B+C-2D$
- x^0: $5 = 4A-4B+D$

$5 = 4A+1 \rightarrow A=1$

(-4) $0 = B+C$

$-3 = 8C-3D$

(-2) $-2 = 4B+C-2D$ $3\leftarrow$

$D=1$

$0 = B+C$ /13

$0 = 12B - 13C$

$0 = 25B \rightarrow B=0$

$C=0$

$$\int \left(\frac{1}{(x-1)^2} + \frac{1}{x^2+4} \right) dx = \frac{(x-1)^{-1}}{-1} + \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + C = \frac{-1}{x-1} + \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + C$$

$$x \neq 1$$

$$\int \frac{1}{x^2+4}\,dx = \int \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{4}\,2\operatorname{arctg}\left(\frac{x}{2}\right) + C$$

$$t = \frac{x}{2}$$

$$dt = \frac{1}{2}dx \,/\cdot 2$$

$$2dt = dx$$