$$\int (x^2 - \sqrt{x})\,dx = \frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c, \quad x \in \langle 0, +\infty)$$

$$\int x^2 \cdot \cos x\,dx = x^2 \sin x - 2\int x \sin x\,dx = x^2 \sin x - 2\left(-x\cos x + \int \cos x\,dx\right) =$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

$x \in \mathbb{R}$

$u = x^2 \quad u' = 2x$

$v' = \cos x \quad v = \sin x$

$u = x \quad u' = 1$

$v' = \sin x \quad v = -\cos x$

$$(u \cdot v)' = u'v + u \cdot v'$$

$$u \cdot v' = (u \cdot v)' - u'v$$

$$\boxed{\int u \cdot v'\,dx = u \cdot v - \int u'v\,dx}$$

$$\int u \, v' = u \cdot v - \int u' v$$

$$\int x \cdot \ln x \, dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int \frac{x^2}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

$$\frac{x^2}{x} = x$$

$u = \ln x \qquad u' = \frac{1}{x}$

$v' = x \qquad v = \frac{x^2}{2}$

$$\int \sin^3 x \cdot \cos x \, dx = \sin^4 x - 3\int \sin^3 x \cos x \, dx \quad / + 3\int \sin^3 x \cos x \, dx$$

$u = \sin^3 x \qquad u' = 3\sin^2 x \cos x$

$v' = \cos x \qquad v = \sin x$

$$4\int \sin^3 x \cos x \, dx = \sin^4 x \quad /:4$$

$$\int \sin^3 x \cos x \, dx = \frac{1}{4}\sin^4 x + c$$

$$\int x e^{3x}\,dx = \frac{1}{3} x e^{3x} - \frac{1}{3}\int e^{3x}\,dx = \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + c$$

$u = x \qquad u' = 1$

$v' = e^{3x} \qquad v = e^{3x}\cdot\frac{1}{3}$

$$\left(e^{3x}\right)' = e^{3x}\cdot 3$$

$$\left(\frac{1}{3}e^{3x}\right)' = \frac{1}{\cancel{3}}e^{3x}\cdot\cancel{3}$$

$$\int e^{3x}\,dx = \frac{1}{3}\int e^{t}\,dt = \frac{1}{3}e^{t} + c = \frac{1}{3}e^{3x} + c$$

$t = 3x$

$dt = 3dx \quad /:3$

$\frac{1}{3}dt = dx$

$$\int \sin^3 x \cos x \, dx = \int t^3 \, dt = \frac{t^4}{4} + c = \frac{\sin^4 x}{4} + c \quad \Big| \quad \sin^3 x = (\sin x)^3$$

$t = \sin x$

$dt = \cos x \, dx$

$$\int x^2 \sin(x^3 - 5) \, dx = \frac{1}{3} \int \sin u \, du = \frac{1}{3}(-\cos u) + c = -\frac{1}{3}\cos(x^3 - 5) + c$$

$u = x^3 - 5$

$du = 3x^2 dx \quad / :3$

$\frac{1}{3} du = x^2 dx$

$$\int \sin(x^3 - 5) \, dx = \int \sin u \frac{1}{3x^2} du$$

$\boxed{u = x^3 - 5}$

$du = 3x^2 dx \quad / : 3x^2$

$\frac{1}{3x^2} du = dx$

$$\int \frac{x-1}{x^2+4}\,dx = \int \frac{x}{x^2+4}\,dx - \int \frac{1}{x^2+4}\,dx = \underline{\frac{1}{2}\ln(x^2+4) - \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + c}$$

$$\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx = \frac{1}{2}\int \frac{ds}{s} = \frac{1}{2}\ln s = \frac{1}{2}\ln(x^2+4) + c$$

$$s = x^2+4$$

$$ds = 2x\,dx$$

$$\int \frac{1}{x^2+4}\,dx = \int \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{4}\operatorname{arctg}\left(\frac{x}{2}\right)\cdot 2 + c$$

$$= \frac{1}{4}2\int \frac{1}{t^2+1}\,dt = 2\,\frac{1}{4}\operatorname{arctg} t + c$$

$$t = \frac{x}{2}$$

$$dt = \frac{1}{2}dx \,/\cdot 2 \rightarrow dx = 2dt$$

$$\int \sin(8x)\,dx = \frac{1}{8}\int \sin t\,dt = \frac{1}{8}(-\cos t) + c = -\frac{1}{8}\cos 8x + c$$

$$t = 8x$$

$$dt = 8dx \quad /:8$$

$$\frac{1}{8}dt = dx$$

$$\int \sin 8x\,dx = -\frac{1}{8}\cos 8x + c$$

$$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + c$$

$$\int (1-3x)^9\,dx = \frac{(1-3x)^{10}}{10}\cdot\left(-\frac{1}{3}\right) + c$$