lok. extrémy:

$$g_3(x,y,z) = x^2 + y^2 + z^2 - xy + x - 2z$$

$$\frac{\partial g_3}{\partial x} = 2x - y + 1 = 0$$

$$\frac{\partial g_3}{\partial y} = 2y - x = 0 \quad / \cdot 2$$

$$3y + 1 = 0 \rightarrow \underline{y = -\frac{1}{3}}$$

$$x = 2y = \underline{-\frac{2}{3}}$$

$$\frac{\partial g_3}{\partial z} = 2z - 2 = 0 \rightarrow \underline{z = 1}$$

$$P = \left[-\frac{2}{3}, -\frac{1}{3}, 1\right]$$

$$\frac{\partial^2 g_3}{\partial x^2} = 2 \qquad \frac{\partial^2 g_3}{\partial y\, \partial x} = -1 \qquad \frac{\partial^2 g_3}{\partial z\, \partial x} = 0$$

$$D_3 = \begin{array}{c|ccc|} & x & y & z \\ x & 2 & -1 & 0 \\ y & -1 & 2 & 0 \\ z & 0 & 0 & 2 \\ & 2 & -1 & 0 \\ & -1 & 2 & 0 \end{array} = 8-2 = \underline{\underline{6}} > 0$$

$$\frac{\partial^2 g_3}{\partial x\, \partial y} = -1 \qquad \frac{\partial^2 g_3}{\partial y^2} = 2 \qquad \frac{\partial^2 g_3}{\partial z\, \partial y} = 0$$

$$\frac{\partial^2 g_3}{\partial x\, \partial z} = 0 \qquad \frac{\partial^2 g_3}{\partial y\, \partial z} = 0 \qquad \frac{\partial^2 g_3}{\partial z^2} = 2$$

$$D_2 = \begin{array}{c|cc|} & x & y \\ x & 2 & -1 \\ y & -1 & 2 \end{array} = 4-1 = 3 > 0$$

$P = \left[-\frac{2}{3}, -\frac{1}{3}, 1\right]$ nastává

lok. MINIMUM

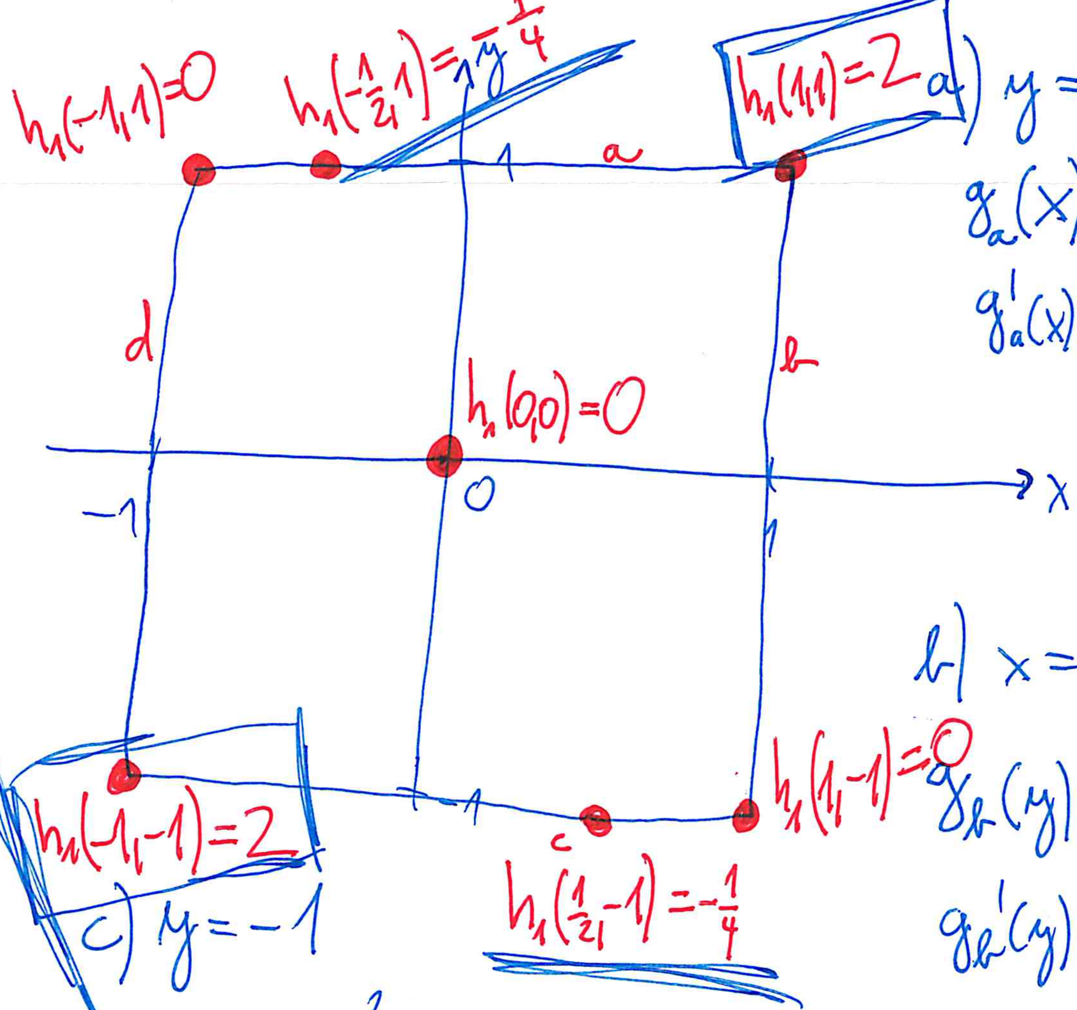
$D_1 = 2 > 0$

$h_1(x,y) = x^2 + xy$ Hledáme extrémy na množině ohraničené čtvercem $[1,1], [1,-1], [-1,1], [-1,-1]$

1) $\dfrac{\partial h_1}{\partial x} = 2x + y = 0$

$\dfrac{\partial h_2}{\partial y} = x = 0$

$P_1 = [0,0]$



a) $y = 1$

$g_a(x) = x^2 + x$ $P_3 = [-\frac{1}{2}, 1]$

$g_a'(x) = 2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

b) $x = 1$ d) $x = -1$

$g_b(y) = 1 + y$ $g_d(y) = 1 - y$

$g_b'(y) = 1 \neq 0$ $g_d'(y) = -1 \neq 0$

$g_c(x) = x^2 - x$

$g_c'(x) = 2x - 1 = 0 \rightarrow x = \frac{1}{2}$ $P_2 = [\frac{1}{2}, -1]$

MAX. JE V $[-1,-1], [1,1]$

$h_1(1,1) = h(-1,-1) = 2$

MIN JE V $[\frac{1}{2}, -1]$ a v $[-\frac{1}{2}, 1]$

$h(x,y) = x^2 + y^2 - xy$

1) $\frac{\partial h}{\partial x} = 2x - y = 0$ $P_1 = [0,0]$

$\frac{\partial h}{\partial y} = 2y - x = 0$

$\downarrow$

$x = 2y \to x = 0$

1.r: $4y - y = 0$

$3y = 0 \to y = 0$

MAX: $[1,0], [0,1], [-1,0], [0,-1]$

$h(1,0) = \ldots = 1$

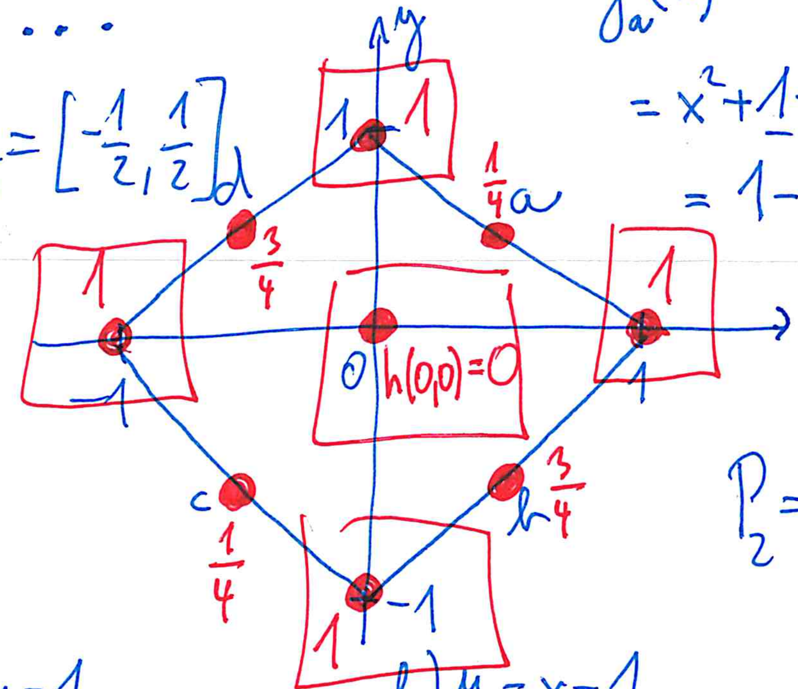
MIN: $[0,0]$

$h(0,0) = 0$

d) $y = x + 1$

...

$P_5 = \left[-\frac{1}{2}, \frac{1}{2}\right]$



a): $y = 1 - x$

$g_a(x) = x^2 + (1-x)^2 - x(1-x) =$

$= x^2 + 1 - 2x + x^2 - x + x^2 =$

$= 1 - 3x + 3x^2$

$g_a'(x) = -3 + 6x = 0$

$6x = 3 \to$

$P_2 = \left[\frac{1}{2}, \frac{1}{2}\right]$ $x = \frac{1}{2}$

$y = \frac{1}{2}$

b) $y = x - 1$

$g_b(x) = x^2 + (x-1)^2 - x(x-1) =$

$= x^2 + x^2 - 2x + 1 - x^2 + x =$

$= x^2 - x + 1$

$g_b'(x) = 2x - 1 = 0 \to x = \frac{1}{2}$

$P_3 = \left[+\frac{1}{2}, -\frac{1}{2}\right]$ $y = -\frac{1}{2}$

c) $y = -x - 1$

$g_c(x) = x^2 + (-x-1)^2 - x(-x-1) =$

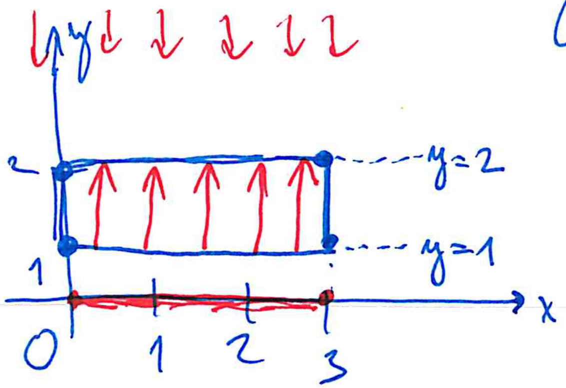
$= x^2 + x^2 + 2x + 1 + x^2 + x =$

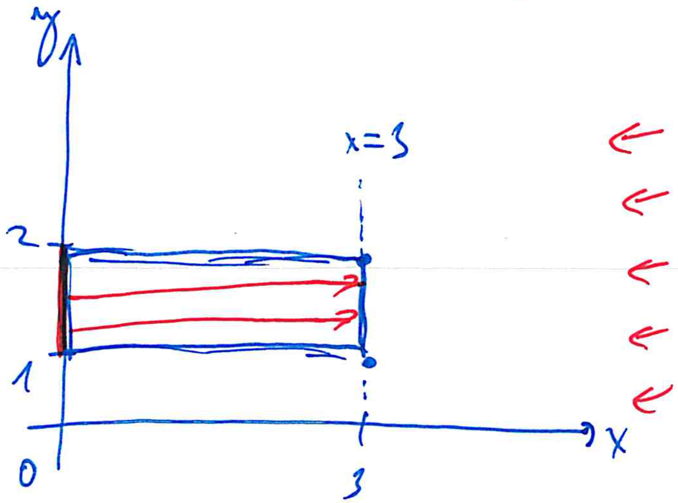
$= 3x^2 + 3x + 1$

$g_c'(x) = 6x + 3 = 0 \to x = -\frac{1}{2}$

$P_4 = \left[-\frac{1}{2}, -\frac{1}{2}\right]$ $y = -\frac{1}{2}$

o) Obdélník s vrholy $[0,1]$, $[3,1]$, $[0,2]$, $[3,2]$





$0 \leq x \leq 3$

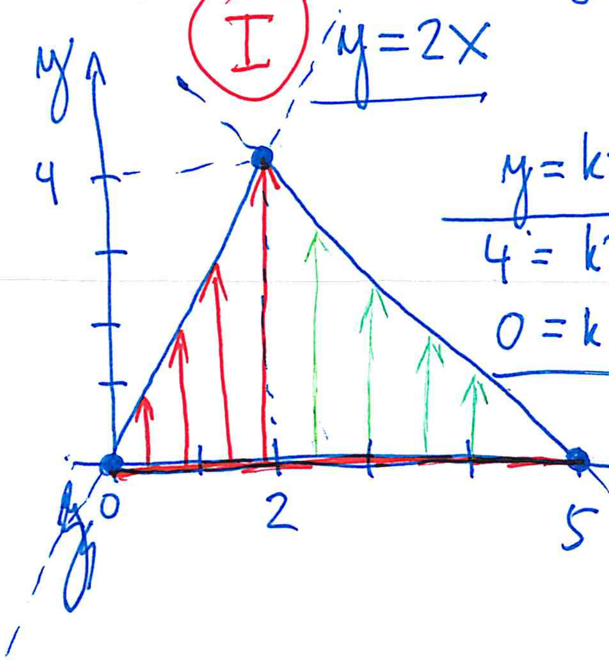
$1 \leq y \leq 2$

$1 \leq y \leq 2$

$0 \leq x \leq 3$

1) A je Δ s vrcholy $[0,0]$, $[2,4]$, $[5,0]$

I



$$y = kx + q$$

$$4 = k2 + q$$

$$0 = k5 + q \quad (-1)$$

$$4 = -3k \rightarrow k = -\frac{4}{3}$$

$$q = 4 + \frac{8}{3} = \frac{20}{3}$$

$$0 \leq x \leq 5$$

$$y = -\frac{4}{3}x + \frac{20}{3} \rightarrow -\frac{4}{3}x = y - \frac{20}{3} \quad / \cdot 3$$

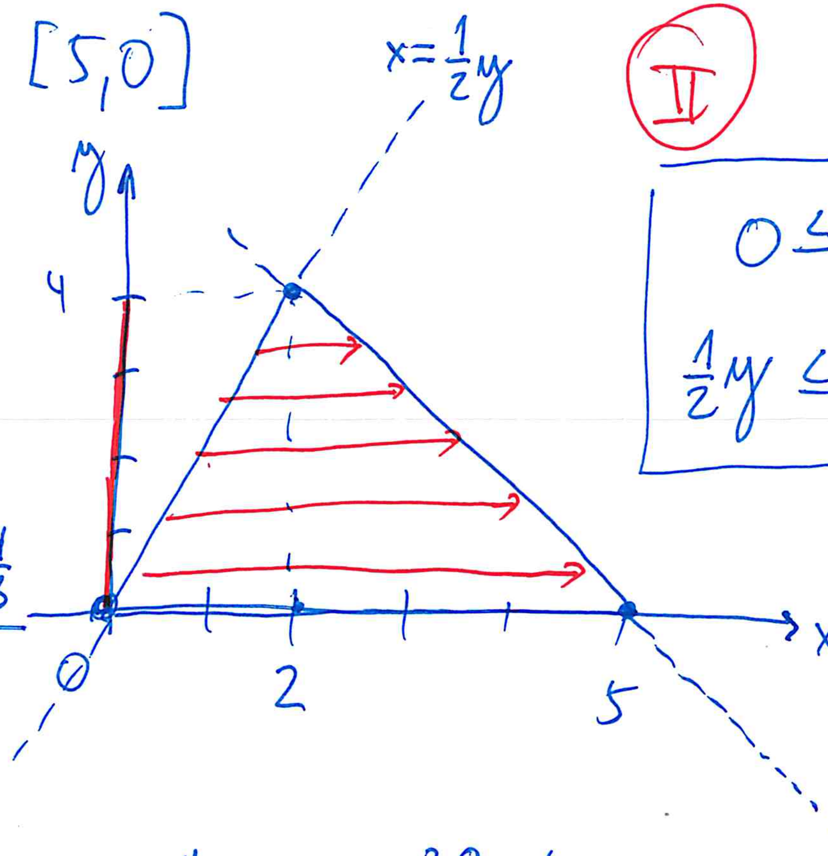
$$-4x = 3y - 20 \quad / : (-4)$$

$$x = -\frac{3}{4}y + 5$$

$$0 \leq x \leq 2 \qquad 2 \leq x \leq 5$$

$$0 \leq y \leq 2x \qquad 0 \leq y \leq -\frac{4}{3}x + \frac{20}{3}$$

II



$$0 \leq y \leq 4$$

$$\frac{1}{2}y \leq x \leq -\frac{3}{4}y + 5$$