$$y' - \frac{2x}{x^2+1} y = 1$$

a) $$y' - \frac{2x}{x^2+1} y = 0$$

$$\frac{dy}{dx} = \frac{2x}{x^2+1} y \quad / : y \quad / \cdot dx$$

$$\int \frac{dy}{y} = \int \frac{2x}{x^2+1} dx \quad \left| \begin{array}{l} t = x^2+1 \\ dt = 2x\,dx \end{array} \right| = \int \frac{1}{t} dt = \ln t + c$$

$$\ln|y| = \ln(x^2+1) + c_1, \quad c_1 \in \mathbb{R}$$

$$|y| = e^{\ln(x^2+1)+c_1} = e^{\ln(x^2+1)} \cdot e^{c_1} = (x^2+1) \cdot c_2$$

$e^{c_1} = c_2 > 0$

$$\underline{y_H = C \cdot (x^2+1), \quad C \in \mathbb{R}}$$

b) $y_p = c\cdot(x^2+1)$, c je teď funkce

$$y_p' = c'(x^2+1) + c\,\cancel{(x^2+1)}\cdot 2x$$

dosazení: $c'(x^2+1) + \cancel{c\cdot 2x} - \cancel{\frac{2x}{x^2+1}\, c\cdot(x^2+1)} = 1$

$$c'(x^2+1) = 1$$

$$c' = \frac{1}{x^2+1} \rightarrow c = \int \frac{1}{x^2+1}\,dx = \operatorname{arctg} x$$

$$y_p = \operatorname{arctg} x\cdot(x^2+1)$$

obecné řešení: $y = y_H + y_p = c(x^2+1) + (x^2+1)\cdot\operatorname{arctg} x = \underline{(x^2+1)(\operatorname{arctg} x + c)}$

$c \in \mathbb{R}$

4) $y' - \frac{1}{x}y = -\ln x \quad , \quad x \in (0, +\infty)$

a) $y' - \frac{1}{x}y = 0$

$$\frac{dy}{dx} = \frac{y}{x} \;/ : y \;/ \cdot dx$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln x + c_1 \quad , c_1 \in \mathbb{R} \quad c_2 > 0$$

$$|y| = e^{\ln x + c_1} = e^{\ln x} \cdot e^{c_1} = c_2 \cdot x$$

$$y_H = c \cdot x \quad , c \in \mathbb{R}$$

b) $y_p = c \cdot x$, c je teď funkce

$$y_p' = c' x + c$$

dosazení: $c' x + \cancel{c} - \frac{1}{\cancel{x}} \cancel{c} \cancel{x} = -\ln x$

$$c' x = -\ln x$$

$$c' = -\frac{\ln x}{x}$$

$$c = -\int \frac{\ln x}{x} dx = -\int t \, dt = -\frac{t^2}{2} = \frac{-\ln^2 x}{2}$$

$t = \ln x$

$dt = \frac{1}{x} dx$

$$y_p = -\frac{\ln^2 x}{2} \cdot x$$

$$y = cx - \frac{\ln^2 x}{2} \cdot x = x\left(c - \frac{\ln^2 x}{2}\right)$$

$c \in \mathbb{R}$

$$y' + 2xy = e^{-x^2}, \quad y(0) = 1$$

a) $y' + 2xy = 0$

$$\frac{dy}{dx} = -2xy \quad \Big/ : y \quad \Big/ \cdot dx$$

$$\int \frac{dy}{y} = \int -2x\,dx$$

$$\ln|y| = -x^2 + c_1, \quad c_1 \in \mathbb{R}$$

$$|y| = e^{-x^2 + c_1} = e^{-x^2} \cdot \underbrace{e^{c_1}}_{c_2 > 0} = c_2 e^{-x^2}$$

$$y_H = c\,e^{-x^2}, \quad \underline{c \in \mathbb{R}}$$

b) $y_p = c\,e^{-x^2}$, c je teď funkce

$$y_p' = c' e^{-x^2} - c\,e^{-x^2} 2x$$

dosazení: $c' e^{-x^2} \cancel{- c\,e^{-x^2} \cdot 2x} \cancel{+ 2x\,c\,e^{-x^2}} = e^{-x^2}$

$$c' \cancel{e^{-x^2}} = \cancel{e^{-x^2}}$$

$$c' = 1$$

$$c = \int 1\,dx = x$$

$c \in \mathbb{R}$

$$y_p = x e^{-x^2} \to y = c e^{-x^2} + x e^{-x^2}$$

$$y = e^{-x^2}(c + x)$$

c) $y(0) = 1$

$$1 = 1(c + 0) \to \underline{c = 1}$$

$$\boxed{y = e^{-x^2}(1 + x)}$$

$$\boxed{y' + x^2 y = x^2, \; y(0) = -1}$$

a) $y' + x^2 y = 0$

$$\frac{dy}{dx} = -x^2 y \quad / : y \quad / \cdot dx$$

$$\int \frac{dy}{y} = \int -x^2 \, dx$$

$$\ln|y| = -\frac{x^3}{3} + c_1, \; c_1 \in \mathbb{R}$$

$$|y| = e^{-\frac{x^3}{3} + c_1} = e^{-\frac{x^3}{3}} \cdot \overset{= c_2 > 0}{e^{c_1}} = c_2 e^{-\frac{x^3}{3}}$$

$$y_H = c \cdot e^{-\frac{x^3}{3}}$$

b) $y_p = c \cdot e^{-\frac{x^3}{3}}$, c je teď ~~pouze~~ funkce

$$y_p' = c' \cdot e^{-\frac{x^3}{3}} - c e^{-\frac{x^3}{3}} x^2$$

dosazení: $c' e^{-\frac{x^3}{3}} - \cancel{c e^{-\frac{x^3}{3}} x^2} + \cancel{x^2 c e^{-\frac{x^3}{3}}} = x^2$

$$c' e^{-\frac{x^3}{3}} = x^2$$

$$c' = e^{\frac{x^3}{3}} x^2$$

$$c = \int e^{\frac{x^3}{3}} x^2 \, dx = \int e^t \, dt = e^t =$$

$$t = \frac{x^3}{3}$$

$$dt = x^2 \, dx$$

$$c = e^{\frac{x^3}{3}}$$

$$y_p = e^{\frac{x^3}{3}} \cdot e^{-\frac{x^3}{3}} = e^0 = 1$$

c) $y = y_H + y_p = c e^{-\frac{x^3}{3}} + 1 \rightarrow -1 = c \cdot e^0 + 1 \rightarrow c = -2$

$$\boxed{y = 1 - 2e^{-\frac{x^3}{3}}}$$

$$y' - 2xy = 2x e^{x^2}, \quad y(0) = 4$$

a) $y' - 2xy = 0$

$$\frac{dy}{dx} = 2xy \quad / : y \; / \cdot dx$$

$$\int \frac{dy}{y} = \int 2x \, dx$$

$$\ln|y| = x^2 + c_1, \quad c_1 \in \mathbb{R}$$

$$|y| = e^{x^2 + c_1} = e^{x^2} \cdot \overbrace{e^{c_1}}^{= c_2 > 0} = c_2 e^{x^2}$$

$$y_H = c e^{x^2}, \quad c \in \mathbb{R}$$

b) $y_p = c e^{x^2}$, c je teď funkce

$$y_p' = c' e^{x^2} + c e^{x^2} \cdot 2x$$

dosazení: $c' e^{x^2} + \cancel{c e^{x^2} \cdot 2x} - \cancel{2x c e^{x^2}} = 2x e^{x^2}$

$$c' \cancel{e^{x^2}} = 2x \cancel{e^{x^2}}$$

$$c' = 2x \to c = x^2$$

$$y_p = x^2 e^{x^2}$$

$$y = c e^{x^2} + x^2 e^{x^2} = e^{x^2}(c + x^2), \quad c \in \mathbb{R}$$

c) $4 = 1(c + 0) \to c = 4$

$$\boxed{y = e^{x^2}(4 + x^2)}$$

$$x^2+2x+7=0$$

$$x_{1,2}=\frac{-2\pm\sqrt{4-28}}{2}=\frac{-2\pm\sqrt{-24}}{2}=\frac{-2\pm\sqrt{24}\cdot\sqrt{-1}}{2}=\frac{-\cancel{2}\pm\cancel{2}\sqrt{6}\,i}{\cancel{2}}=\underline{-1\pm\sqrt{6}\,i}$$

$x_1=-1+\sqrt{6}\,i$

$x_2=-1-\sqrt{6}\,i$

číslo i : $\underline{i^2=-1}$ $\quad i=\sqrt{-1}$

$z_1=3+2i,\ z_2=5-7i,\ z_3=\sqrt{2}+\sqrt{3}\,i$

$z=a+bi$

$$x^2-2x+2=0$$

$$x_{1,2}=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm\sqrt{-4}}{2}=\frac{2\pm\sqrt{4}\sqrt{-1}}{2}=\frac{2\pm 2i}{2}=\underline{1\pm i}\ \begin{cases}\underline{1+i}\\ \underline{1-i}\end{cases}$$

zk: $(1+i)^2-2(1+i)+2\overset{?}{=}0$

$1+\cancel{2i}+\underbrace{i^2}_{-1}\underline{-2}-\cancel{2i}\underline{+2}=0$ ✓

$x^4 - \cancel{} 16 = 0$ $\quad x^4 = 16$: $x_1 = 2$, $x_2 = -2$, $x_3 = 2i$, $x_4 = -2i$

$(x^2 - 4)(x^2 + 4) = 0$

$(2i)^4 = 2^4 \cdot i^4 = 16 i^2 \cdot i^2 = 16 \cdot (-1)(-1) = 16$

$(x-2)(x+2)(x^2+4) = 0$

$\downarrow$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm\sqrt{4}\sqrt{-1} = \pm 2i$

$(x-2)(x+2)(x-2i)(x+2i) = 0$