$$y'''' - 16y = 0$$

$\downarrow$

charakteristická rovnice

$$\lambda^4 - 16 = 0 \qquad \lambda^4 = 16$$

$$\boxed{\text{očekávali jsme řešení } y = e^{\lambda x}}$$

$$(\lambda^2 - 4)(\lambda^2 + 4) = 0$$

$$\lambda^2 = -4 = 4(-1)$$

$$\lambda = \pm\sqrt{4}\sqrt{-1} = \pm 2i$$

$$\boxed{\begin{aligned} \lambda_{1,2} = a \pm bi &\to y_1 = e^{ax}\cos bx \\ &\to y_2 = e^{ax}\sin bx \end{aligned}}$$

$$(\lambda - 2)(\lambda + 2)(\lambda - 2i)(\lambda + 2i) = 0$$

$$\lambda_1 = 2 \quad \lambda_2 = -2 \quad \lambda_3 = 2i \quad \lambda_4 = -2i$$

$$y_1 = e^{2x} \qquad y_2 = e^{-2x}$$

$$\cancel{y_3 = e^{2ix}} \to y_3 = e^{0x}\cos 2x = \cos 2x$$

$$\cancel{y_4 = e^{-2ix}} \qquad y_4 = e^{0x}\sin 2x = \sin 2x$$

obecné řešení:

$$\boxed{y = c_1 e^{2x} + c_2 e^{-2x} + c_3\cos 2x + c_4\sin 2x} \qquad c_1, c_2, c_3, c_4 \in \mathbb{R}$$

$$y''' - 3y'' + 3y' - y = 0$$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$(\lambda - 1)^3 = 0$$

$$\lambda_{1,2,3} = 1$$

$$y_1 = e^x, \quad y_2 = xe^x, \quad y_3 = x^2 e^x$$

$$y = c_1 e^x + c_2 x e^x + c_3 x^2 e^x, \quad c_1, c_2, c_3 \in \mathbb{R}$$

$$y^{(6)} - y^{(4)} - 2y^{(2)} = 0$$

$$\lambda^6 - \lambda^4 - 2\lambda^2 = 0$$

$$(\lambda^4 - \lambda^2 - 2)\cdot\lambda^2 = 0$$

$$\lambda^2 = 0 \to \boxed{\lambda_{1,2} = 0} \to y_1 = e^{0x} = 1,\ y_2 = xe^{0x} = x$$

$$(\lambda - 0)(\lambda - 0) = 0$$

$$\lambda^4 - \lambda^2 - 2 = 0$$

$$\boxed{t = \lambda^2}$$

$$t^2 = \lambda^4$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$t_1 = 2 \to \lambda^2 = 2 \quad \boxed{\lambda_{3,4} = \pm\sqrt{2}} \to y_3 = e^{\sqrt{2}x},\ y_4 = e^{-\sqrt{2}x}$$

$$t_2 = -1 \quad \lambda^2 = -1 \quad \boxed{\lambda_{5,6} = \pm i} \quad (\overset{a}{0} \pm \overset{b}{1}i)$$

$$(\lambda^2 - 2)(\lambda^2 + 1) = 0$$

$$y_5 = e^{0x}\cos x,\ y_6 = e^{0x}\sin x$$

$$\boxed{y = c_1 + c_2 x + c_3 e^{\sqrt{2}x} + c_4 e^{-\sqrt{2}x} + c_5 \cos x + c_6 \sin x}$$

$c_1, c_2, c_3, c_4, c_5, c_6 \in \mathbb{R}$

$$y'' + y' - 2y = 3xe^{x}$$

1) $y'' + y' - 2y = 0$

$$\lambda^2 + \lambda - 2 = 0$$

$$(\lambda+2)(\lambda-1) = 0$$

$$\lambda_1 = -2 \quad \lambda_2 = 1$$

$$y_1 = e^{-2x} \quad y_2 = e^{x}$$

$$y_H = c_1 e^{-2x} + c_2 e^{x}, \; c_1, c_2 \in \mathbb{R}$$

$1x \to k=1$

2) $y'' + y' - 2y = e^{1x} \cdot 3x \cdot \cos 0x$

$\alpha = 1$

$\beta = \cancel{1}0$

$\alpha + i\beta = 1 + 0i = 1$ je jednonásobný kořen ch. rovnice

$\downarrow$

$k = 1$

V zadání je polynom 1. stupně, $R_n(x)$ a $S_n(x)$ jsou polynomy 1. stupně

$$y_p = e^{\alpha x} \cdot x^{k} \left( R_n(x) \cos\beta x + S_n(x) \sin\beta x \right)$$

$\downarrow$

$$y_p = e^{x} \cdot x \left( (Ax+B) \underbrace{\cos 0x}_{1} + \cancel{(Cx+D) \underbrace{\sin 0x}_{0}} \right)$$

$$y_p = e^{x} (Ax^2 + Bx)$$

$$y'' + y' - 2y = 3xe^x$$

$$\left.\begin{aligned} y_p &= (Ax^2 + Bx)e^x \\ y_p' &= (2Ax+B)e^x + (Ax^2+Bx)e^x = (Ax^2+2Ax+Bx+B)e^x \\ y_p'' &= (2Ax+2A+B)e^x + (Ax^2+2Ax+Bx+B)e^x \end{aligned}\right\}$$ dosadíme do rovnice

$$(2Ax+2A+B)\cancel{e^x} + (\cancel{Ax^2}+2Ax+Bx+B)\cancel{e^x} + (\cancel{Ax^2}+2Ax+Bx+B)\cancel{e^x} - 2(\cancel{Ax^2}+Bx)\cancel{e^x} = 3x\cancel{e^x}$$

u $x$: $6A = 3 \rightarrow A = \frac{1}{2}$

u $x^0$: $2A+B+B+B = 0 \rightarrow 3B = -2A = -1 \rightarrow B = -\frac{1}{3} \rightarrow \boxed{y_p = \left(\frac{1}{2}x^2 - \frac{1}{3}x\right)e^x}$

obecné řešení: $y = y_H + y_P$

$$y = c_1 e^{-2x} + c_2 e^x + \left(\frac{1}{2}x^2 - \frac{1}{3}x\right)e^x, \quad c_1, c_2 \in \mathbb{R}$$

Špatně: $y'' + y' - 2y = 3xe^x$

$$y_p = (Ax+B)e^x$$

$$y_p' = Ae^x + (Ax+B)e^x = (Ax+A+B)e^x$$

$$y_p'' = Ae^x + (Ax+A+B)e^x$$

dosazení: $A\cancel{e^x} + \underbrace{(Ax+A+B)}_{''}\cancel{e^x} + \underbrace{(Ax+A+B)}_{''}\cancel{e^x} - 2(Ax+B)\cancel{e^x} = 3x\cancel{e^x}$

$u\,x: A + A - 2A = 3$

$$0 = 3 \quad \emptyset \rightarrow \text{máme špatně } y_p$$