$$y''' - 3y'' + 3y' - y = e^x$$

1) $y''' - 3y'' + 3y' - y = 0$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$(\lambda - 1)^3 = 0$$

$$\lambda_{1,2,3} = 1$$

$$y_H = c_1 e^x + c_2 e^x \cdot x + c_3 e^x x^2$$

2) $y''' - 3y'' + 3y' - y = e^x$

$\alpha = 1 \quad \beta = 0 \quad k = 3$

$\alpha + i\beta = 1 + 0i = 1$ $3x$

$$y_p = e^x \cdot x^3 (A\cos 0x + B\sin 0x) = Ax^3e^x$$

$$y_p = Ax^3e^x$$

$$y_p' = A3x^2e^x + Ax^3e^x = Ae^x(3x^2 + x^3)$$

$$y_p'' = Ae^x(\underline{3x^2} + x^3) + Ae^x(\underline{6x} + \underline{3x^2}) =$$

$$= Ae^x(6x + 6x^2 + x^3)$$

$$y_p''' = Ae^x(\underline{6x} + \underline{6x^2} + x^3) + Ae^x(\underline{6} + \underline{12x} + \underline{3x^2})$$

$$= Ae^x(6 + 18x + 9x^2 + x^3)$$

dosazení: $A\cancel{e^x}(6+18x+9x^2+x^3) - 3A\cancel{e^x}(6x+6x^2+x^3) + 3A\cancel{e^x}(3x^2+x^3) - Ax^3\cancel{e^x} = \underset{1}{\cancel{e^x}}$

$$\boxed{6A = 1} \longrightarrow A = \frac{1}{6}$$

$$y_p = \frac{1}{6}x^3 e^x$$

obecné řešení: $y = y_H + y_p = c_1 e^x + c_2 x e^x + c_3 x^2 e^x + \frac{1}{6}x^3 e^x$

$c_1, c_2, c_3 \in \mathbb{R}$

Řešení Cauchyovy úlohy pomocí Laplaceovy tranformace

$$y' - 2y = 1\,, \quad y(0) = -2$$

$$\mathcal{L}(1) = \frac{1}{s}$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y') = sY - (-2) = sY + 2$$

obraz rovnice: $\underbrace{sY + 2}_{y''} \underbrace{-2Y}_{-2y} = \underbrace{\frac{1}{s}}_{1} \quad / -2$

$$Y(s-2) = \frac{1}{s} - 2 \quad /:(s-2)$$

$$Y = \frac{1}{\underbrace{s(s-2)}_{a=0 \quad b=-2}} - \frac{2}{s-2} \longrightarrow y = \frac{1 - e^{2t}}{-2} - 2e^{2t} = -\frac{1}{2} - \frac{3}{2}e^{2t}$$

$$y'' + 4y = 4e^{-2t}, \quad y(0) = 1, \quad y'(0) = 0$$

$$\mathcal{L}(e^{-2t}) = \frac{1}{s+2}$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y') = sY - 1$$

$$\mathcal{L}(y'') = s^2 Y - s \cdot 1 - 0$$

obraz rownie: $s^2 Y - s + 4Y = \frac{4}{s+2} \quad / +s$

$$Y(s^2+4) = \frac{4}{s+2} + s \quad / : (s^2+4)$$

$$Y = \frac{4}{(s+2)(s^2+4)} + \frac{s}{s^2+4} = \frac{4 + s(s+2)}{(s+2)(s^2+4)} = \frac{s^2+2s+4}{(s+2)(s^2+4)}$$

$\cos 2t$

$$\frac{4}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4} = \frac{A(s^2+4)+(Bs+C)(s+2)}{(s+2)(s^2+4)} = \frac{As^2+4A+Bs^2+2Bs+Cs+2C}{(s+2)(s^2+4)}$$

u s^2: $0 = A + B \rightarrow A = -B$

u s: $0 = 2B + C$ $\quad /\cdot 2$

u s^0: $4 = 4A + 2C = -4B + 2C$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} 4 = 4C \rightarrow \\ 2B = -1 \rightarrow \end{matrix} \boxed{\begin{matrix} C = 1 \\ B = -\frac{1}{2} \\ A = \frac{1}{2} \end{matrix}}$$

$$Y = \frac{4}{(s+2)(s^2+4)} + \frac{s}{s^2+4} = \frac{\frac{1}{2}}{s+2} + \frac{\frac{1}{2}s}{s^2+4} + \frac{1}{s^2+4} + \frac{s}{s^2+4}$$

$$y = \frac{1}{2}e^{-2t} + \frac{1}{2}\cos 2t + \frac{1}{2}\sin 2t + \cos 2t = \frac{1}{2}e^{-2t} + \frac{3}{2}\cos 2t + \frac{1}{2}\sin 2t$$

$$y'' + y' + 4y = \cos 2t, \quad y(0) = 0, \quad y'(0) = 1$$

$$\mathcal{L}(\cos 2t) = \frac{s}{s^2+4}$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y') = sY - 0$$

$$\mathcal{L}(y'') = s^2Y - s \cdot 0 - 1$$

obraz rovnice: $s^2Y - 1 + sY + 4Y = \dfrac{s}{s^2+4} \quad \Big/ +1$

$$Y(s^2+s+4) = \frac{s}{s^2+4} + 1 \quad \Big/ :(s^2+s+4)$$

$$Y = \frac{s}{(s^2+4)(s^2+s+4)} + \frac{1}{s^2+s+4} = \frac{\cancel{s+s^2+4}}{(s^2+4)\cancel{(s^2+s+4)}} = \frac{1}{s^2+4}$$

$$y = \frac{1}{2}\sin 2t$$

$$y''' + y' = e^{2t} \qquad y(0) = y'(0) = y''(0) = 0$$

$$\mathcal{L}(e^{2t}) = \frac{1}{s-2}$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y') = sY$$

$$\mathcal{L}(y''') = s^3 Y - s^2 \cdot 0 - s \cdot 0 - 0$$

obraz rovnice:

$$s^3 Y + sY = \frac{1}{s-2}$$

$$Y(s^3+s) = \frac{1}{s-2} \quad / : (s^3+s)$$

$$Y = \frac{1}{(s-2)(s^3+s)} = \frac{1}{(s-2)(s^2+1)s}$$

$$Y = \frac{1}{(s-2)(s^2+1)\,s} = \frac{A}{s} + \frac{B}{s-2} + \frac{Cs+D}{s^2+1} = \frac{A(s-2)(s^2+1) + Bs(s^2+1) + (Cs+D)(s-2)\cdot s}{s(s-2)(s^2+1)}$$

$$= \frac{A(s^3-2s^2+s-2) + B(s^3+s) + Cs^3 - 2Cs^2 + Ds^2 - 2Ds}{s(s-2)(s^2+1)}$$

$$u\ s^3:\ A + B + C = 0$$

$$s^2:\ -2A - 2C + D = 0$$

$$s:\ A + B - 2D = 0$$

$$s^0:\ -2A = 1 \ \rightarrow\ A = -\frac{1}{2}$$

$$\left.\begin{array}{l} B + C = \frac{1}{2} \\ \left.\begin{array}{l} -2C + D = -1 \quad /\cdot 2 \\ B - 2D = \frac{1}{2} \end{array}\right\} B - 4C = -\frac{3}{2} \end{array}\right\} 5B = \frac{1}{2}$$

$/\cdot 4$

$$B = \frac{1}{10}$$

$$C = \frac{2}{5}$$

$$2D = \frac{1}{2} - B = \frac{1}{2} - \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

$$D = \frac{2}{10}$$

$$Y = \frac{-\frac{1}{2}}{s} + \frac{\frac{1}{10}}{s-2} + \frac{\frac{2}{5}s}{s^2+1} + \frac{\frac{2}{10}}{s^2+1}$$

$$y = -\frac{1}{2} + \frac{1}{10}e^{2t} + \frac{2}{5}\cos t + \frac{2}{10}\sin t$$

$$y'' + 4y' + 8y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

$$\mathcal{L}(0) = 0$$

$$\mathcal{L}(y) = Y$$

$$\mathcal{L}(y') = sY - 0$$

$$\mathcal{L}(y'') = s^2Y - 0s - 1$$

obraz rovnice:

$$s^2Y - 1 + 4sY + 8Y = 0 \quad /+1$$

$$Y(s^2 + 4s + 8) = 1$$

$$Y = \frac{1}{s^2+4s+8} = \frac{1}{(s+2)^2+4} \rightarrow y = \frac{1}{2}e^{-2t}\sin 2t$$