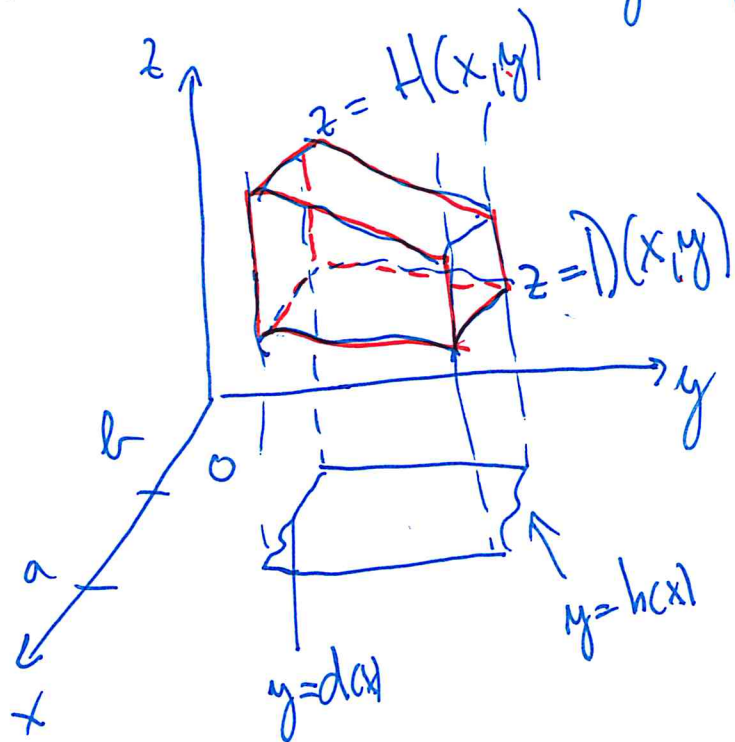


Integrační obory pro trojný integrál

težera ohraničená grafy spojitých funkcí

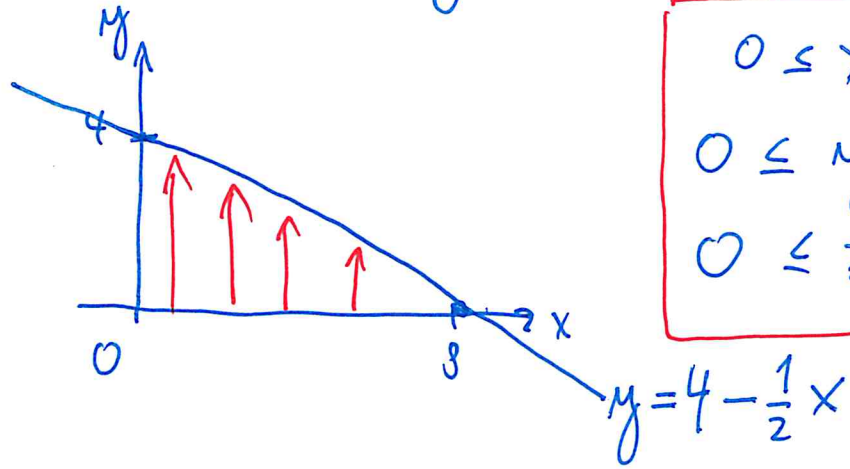
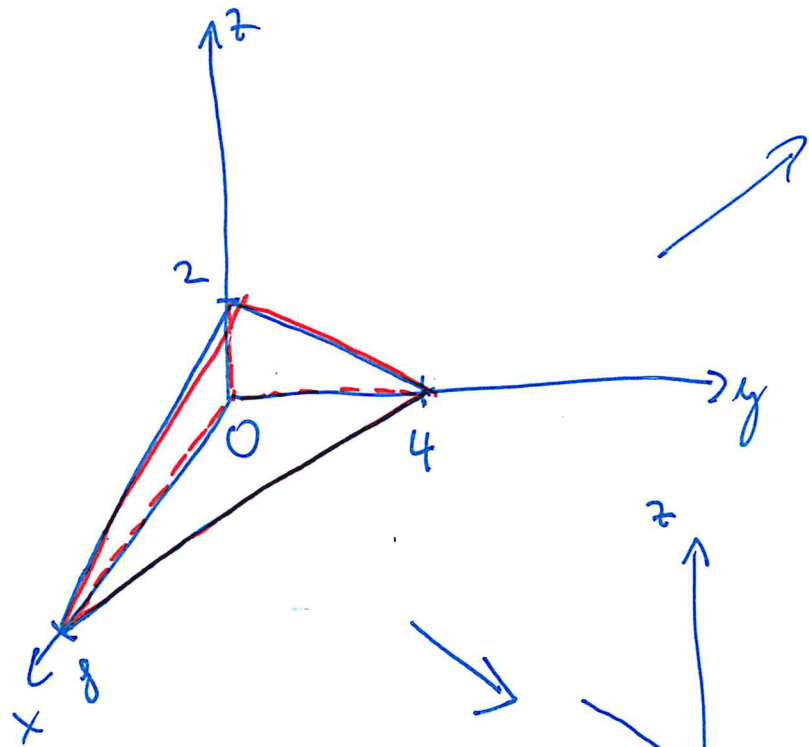


$$a \leq x \leq b$$

$$d(x) \leq y \leq h(x)$$

$$D(x,y) \leq z \leq H(x,y)$$

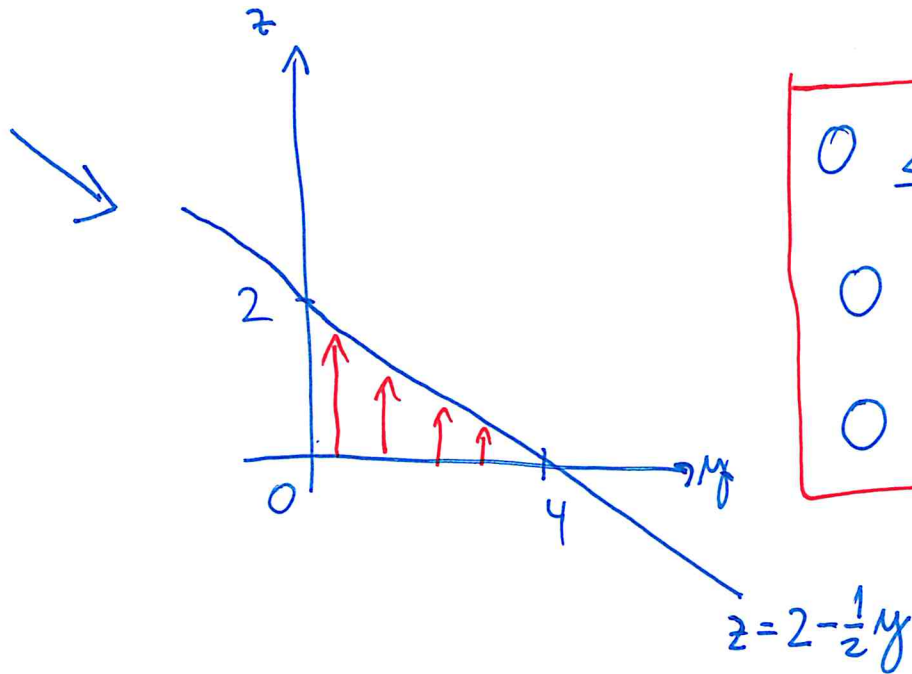
Těleso ohraničené $x=0, y=0, z=0$ a $x+2y+4z=8$



$$0 \leq x \leq 8$$

$$0 \leq y \leq 4 - \frac{1}{2}x$$

$$0 \leq z \leq z = \frac{8 - 2y - x}{4}$$



$$0 \leq y \leq 4$$

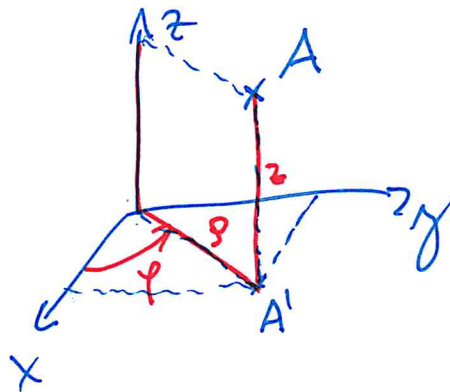
$$0 \leq z \leq 2 - \frac{1}{2}y$$

$$0 \leq x \leq 8 - 2y - 4z$$

Válcové souřadnice

$$\rho, \varphi, z$$

ρ a φ jsou polární souřadnice průmětu bodu do roviny xy



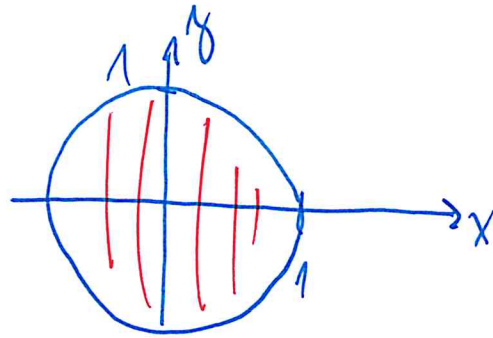
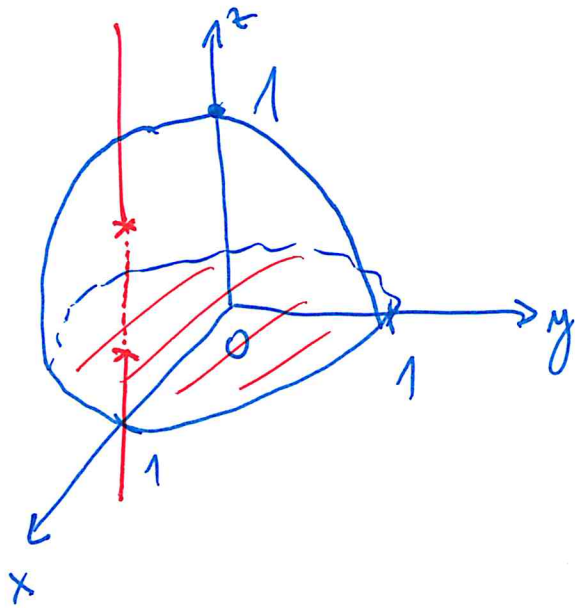
$$x = \rho \cos \varphi$$

$$y = \rho \sin \varphi$$

$$z = z$$

$$J = \rho$$

Těleso ohraničené $z = 1 - x^2 - y^2$ a $z = 0$:



$$x = \rho \cos \varphi$$

$$y = \rho \sin \varphi$$

$$z = z$$

$$z = 1 - x^2 - y^2$$

$$z = 1 - \rho^2 \cos^2 \varphi - \rho^2 \sin^2 \varphi$$

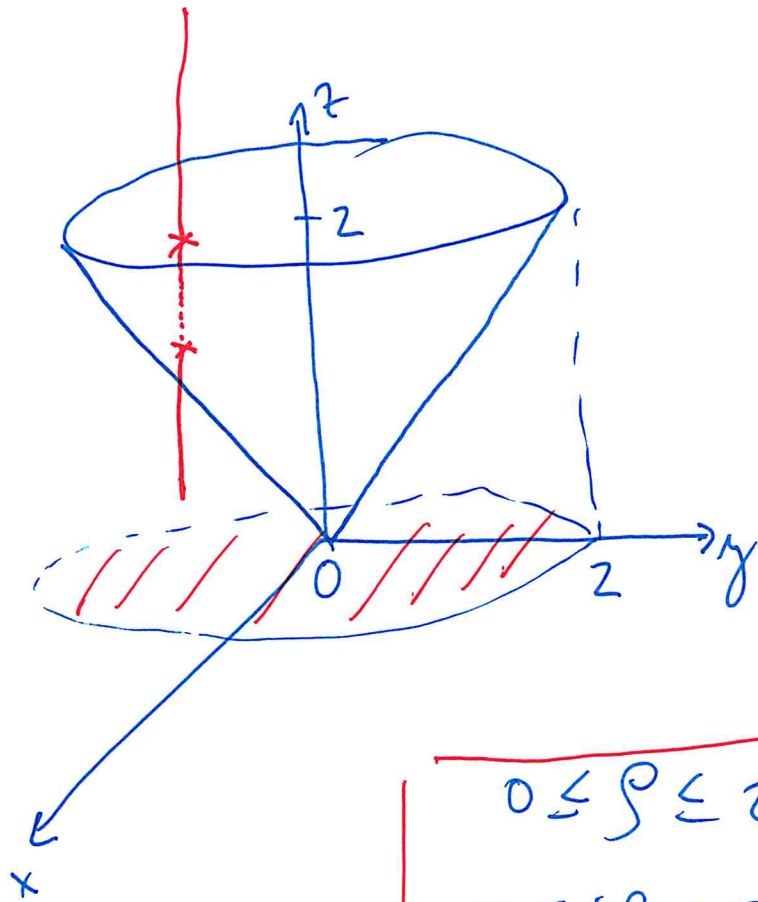
$$z = 1 - \rho^2$$

$$0 \leq \rho \leq 1$$

$$0 \leq \varphi \leq 2\pi$$

$$0 \leq z \leq 1 - \rho^2$$

Tēluro ohrani cēnē $z = \sqrt{x^2 + y^2}$ a $z = 2$



$$y=0: z = \sqrt{x^2} = |x|$$

$$x=0: z = \sqrt{y^2} = |y|$$

$$z=2 \quad \sqrt{x^2 + y^2} = 2$$

$$x^2 + y^2 = 4$$

$$z = \sqrt{x^2 + y^2} = \sqrt{\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi} =$$

$$= \sqrt{\rho^2} = |\rho| = \rho \quad (\rho \geq 0)$$

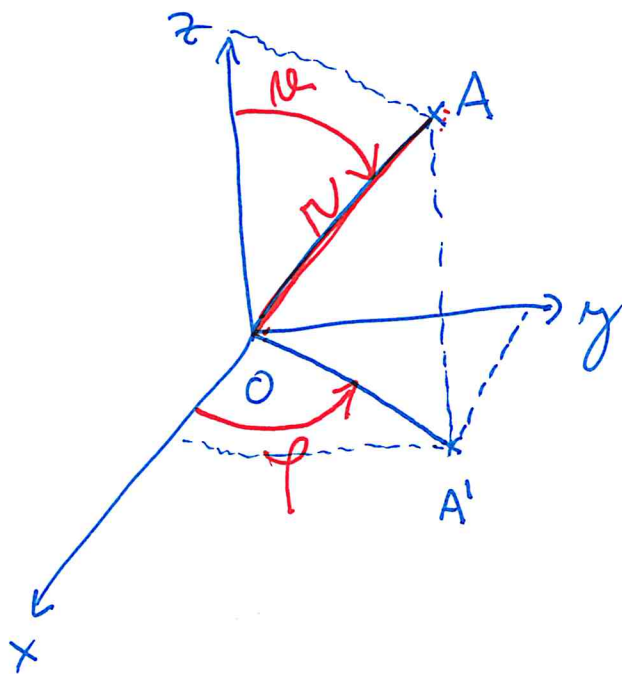
$$\begin{aligned} 0 &\leq \rho \leq 2 \\ 0 &\leq \varphi \leq 2\pi \\ \rho &\leq z \leq 2 \end{aligned}$$

Sférické (kulové) souřadnice

sféra : plocha $x^2 + y^2 + z^2 = R^2$

koule : těleso $x^2 + y^2 + z^2 \leq R^2$

souřadnice r, φ, ϱ



r - vzdálenost bodu od počátku ($r \geq 0$)

φ - orientovaný úhel, který svírá
přímka přivodící do xy s

kladnou poloosou x $\varphi \in \langle 0, 2\pi \rangle$

ϱ - orientovaný úhel, který svírá přivodící
s kladnou poloosou z $\varrho \in \langle 0, \pi \rangle$

$$x = r \sin \varrho \cos \varphi$$

$$y = r \sin \varrho \sin \varphi$$

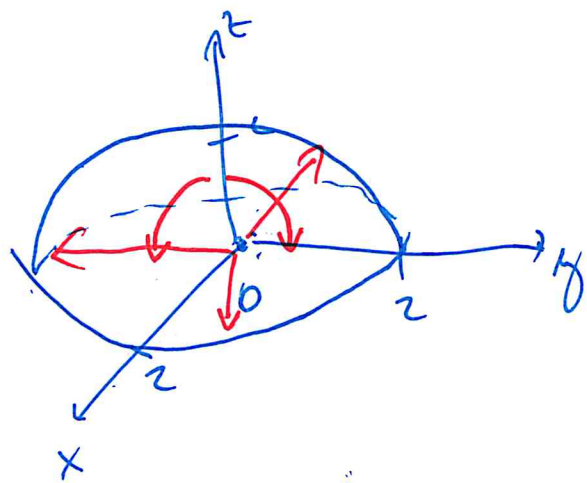
$$z = r \cos \varrho$$

$$J = r^2 \sin \varrho$$

Těleso je popsáno

$$x^2 + y^2 + z^2 \leq 4, z \geq 0$$

koní polokoule

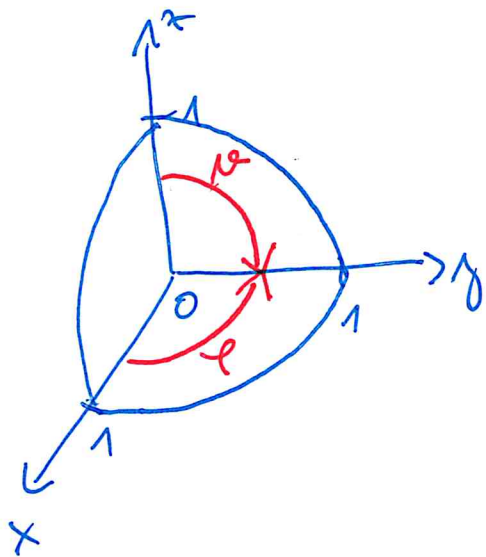


$$0 \leq r \leq 2$$

$$0 \leq \varphi \leq 2\pi$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

Těleso $x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0, z \geq 0$



$$0 \leq r \leq 1$$

$$0 \leq \varphi \leq \frac{\pi}{2}$$

$$0 \leq \theta \leq \frac{\pi}{2}$$