$$\int \sin 4x \, dx = \frac{1}{4}\int \sin t \, dt = -\frac{1}{4}\cos t + c = -\frac{1}{4}\cos 4x + c$$

$$c \in R, \; x \in R$$

$t = 4x$

$dt = 4\,dx \quad /:4$

$\frac{1}{4}dt = dx$

$$\int \sin 4x \, dx = -\frac{1}{4}\cos 4x + c$$

$$\int \sqrt[3]{1-5x}\,dx = \int (1-5x)^{\frac{1}{3}}\,dx = \frac{(1-5x)^{\frac{4}{3}}}{\frac{4}{3}} \cdot \left(-\frac{1}{5}\right) + c$$

$$c \in R, \; x \in R$$

$t = 1-5x$

$$\int \frac{1}{2-5x}\,dx = \left(-\frac{1}{5}\right)\ln|2-5x| + c$$

$$c \in R$$

$$x \neq \frac{2}{5}$$

$$\int \frac{1}{(3x-1)^4}\,dx = \int (3x-1)^{-4}\,dx = \frac{(3x-1)^{-3}}{-3}\cdot\frac{1}{3} + c$$

$$x \neq \frac{1}{3},\ c \in \mathbb{R}$$

$$\int \underline{\sin^4 x}\cdot \cos x\, dx = \int t^4\,dt = \frac{t^5}{5} + c = \frac{(\sin x)^5}{5} + c$$

$t = \sin x$

$dt = \cos x\, dx$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$$\int 3x \cdot e^{\boxed{x^3+5}}\,dx = \frac{3}{2}\int e^{v}\,dv = \frac{3}{2}e^{v} + c = \frac{3}{2}e^{x^2+5} + c$$

$v = x^2+5$

$dv = 2x\,dx$

$\frac{1}{2}dv = x\,dx$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$$\int \frac{2x-2}{x^2-2x+7}\,dx = \int \frac{1}{s}\,ds = \int \frac{ds}{s} = \ln|s| + c =$$

$s = x^2-2x+7$

$ds = (2x-2)\,dx$

$$= \ln(x^2-2x+7) + c \qquad c \in \mathbb{R},\ x \in \mathbb{R}$$

$$\int \frac{x-2}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx - \int \frac{2}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4) - \operatorname{arctg}\left(\frac{x}{2}\right) + c$$

$*$

$$\int \frac{2x}{x^2+4}\,dx = \int \frac{1}{t}\,dt = \ln|t| + c = \ln(x^2+4) + c$$

$c \in \mathbb{R}$

$x \in \mathbb{R}$

$t = x^2+4$

$dt = 2x\,dx$

$$\int \frac{2}{x^2+4}\,dx = 2\int \frac{1}{x^2+4}\,dx = 2\int \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx = \frac{1}{2}\int \frac{1}{\frac{x^2}{2^2}+1}\,dx =$$

$$= \frac{1}{2}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{\cancel{2}}\cancel{2}\,\operatorname{arctg}\left(\frac{x}{2}\right)^{*} + c$$

$x \in \mathbb{R}$

$c \in \mathbb{R}$

$s = \frac{x}{2}$

$ds = \frac{1}{2}dx \rightarrow dx = \underline{2}ds$

$$\int \frac{2x+3}{x^2+2x+2}\,dx = \int \frac{2x+2}{x^2+2x+2}\,dx + \int \frac{1}{x^2+2x+2}\,dx$$

$$\int \frac{2x+2}{\underbrace{x^2+2x+2}}\,dx = \int \frac{1}{r}\,dr = \ln(r) + c = \ln(x^2+2x+2) + c$$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$r = \underbrace{x^2+2x+2}$

$dr = (2x+2)\,dx$

$$\int \frac{1}{x^2+2x+2}\,dx = \int \frac{1}{(x+1)^2+1}\,dx = \operatorname{arctg}(x+1) + c$$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$(a+b)^2 = a^2 + 2ab + b^2$

$x^2+2x+2 = \underbrace{x^2+2x+1}+1 = (x+1)^2+1$

$$\int x^2 \cdot \sqrt[4]{x^3-5}\,dx = \frac{1}{3}\int \sqrt[4]{t}\,dt = \frac{1}{3}\,\frac{t^{\frac{5}{4}}}{\frac{5}{4}} + C =$$

$$\sqrt[4]{t} = t^{\frac{1}{4}}$$

$$t = x^3 - 5$$

$$dt = 3x^2\,dx \quad /:3$$

$$\frac{1}{3}\,dt = x^2\,dx$$

$$= \frac{4}{15}\left(x^3-5\right)^{\frac{5}{4}} + C$$

$C \in \mathbb{R}$

$$x^3 - 5 > 0$$

$$x^3 > 5$$

$$x > \sqrt[3]{5}$$

$$\int \frac{e^x}{e^x+5}\,dx = \int \frac{1}{t}\,dt = \ln|t| = \ln(e^x+5) + c$$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$t = e^x + 5$

$dt = e^x\,dx$

$$\int \frac{f'(t)}{f(t)}\,dt = \int \frac{1}{r}\,dr = \ln|r| + c = \boxed{\ln|f(t)| + c = \int \frac{f'(t)}{f(t)}\,dt}$$

$r = f(t)$

$dr = f'(t)\,dt$

$$\int \frac{e^x}{e^x+5}\,dx = \int \frac{dr}{r+5} = \ln|r+5| + c = \ln(e^x+5) + c$$

$r = e^x$

$dr = e^x\,dx$

$$\int \sin x \cdot \cos x\,dx = \int t\,dt = \frac{t^2}{2} + c = \frac{\sin^2 x}{2} + c$$

$t = \sin x$

$dt = \cos x\,dx$

$$\int \sin x \cos x\,dx = \sin^2 x - \int \sin x \cos x\,dx \rightarrow 2\int \sin x \cos x\,dx = \sin^2 x \quad /:2$$

$$\int \sin x \cos x\,dx = \frac{\sin^2 x}{2} + c$$

$\cancel{u = \sin x}$ $\quad u' = \cos x$

$v' = \cos x$ $\quad \cancel{v = \sin x}$

$$\int \sin x \cdot \cos x \, dx = \int \frac{1}{2} \sin 2x \, dx = \frac{1}{2}\left(-\cos 2x \cdot \frac{1}{2}\right) + c = -\frac{1}{4}\underline{\cos 2x} + c =$$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$= -\frac{1}{4}\left(1 - 2\sin^2 x\right) + c =$$

$$= \frac{\sin^2 x}{2} - \frac{1}{4} + c$$

$$\int \frac{1}{x^2 + 4x + 5} \, dx = \int \frac{1}{(x+2)^2 + 1} \, dx = \underline{\operatorname{arctg}(x+2) + c}$$