1)

$$\frac{5x^{1}-2}{x^{2}-4} = \frac{5x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{(x-2)(x+2)} =$$

$$= \frac{Ax+Bx+2A-2B}{(x-2)(x+2)} = \frac{\overbrace{(A+B)}^{5}x \overbrace{+2A-2B}^{-2}}{(x-2)(x+2)} = \underline{\frac{2}{x-2} + \frac{3}{x+2}}$$

u x: $A+B=5 \quad / \cdot 2$

u x^0: $2A-2B=-2$

$4A = 8 \rightarrow A = 2$

$B = 3$

$$\int \frac{5x-2}{x^2-4}\,dx = \int\left(\frac{2}{x-2} + \frac{3}{x+2}\right)dx = 2\ln|x-2| + 3\ln|x+2| + c$$

$c \in \mathbb{R}$

$x \neq \pm 2$

2)
$$\frac{x^4+x^3+12x^2+10x+28}{x^2+9} = x^2+x+3+\frac{x+1}{x^2+9}$$

$$(x^4+x^3+12x^2+10x+28):(x^2+9) = x^2+x+3$$

$$
\begin{array}{l}
-(x^4 \qquad +9x^2) \\
\hline
\quad x^3+3x^2+10x+28 \\
\quad -(x^3 \qquad +9x) \\
\hline
\qquad 3x^2+x+28 \\
\qquad -(3x^2 \qquad +27) \\
\hline
\qquad \boxed{x+1}
\end{array}
$$

$$\int \left(x^2+x+3+\frac{x+1}{x^2+9}\right)dx =$$

$$= \frac{x^3}{3}+\frac{x^2}{2}+3x+\int \frac{x+1}{x^2+9}\,dx$$

$$\int \frac{x+1}{x^2+9}\,dx = \int \frac{x}{x^2+9}\,dx + \int \frac{1}{x^2+9}\,dx$$

$$\int \frac{x}{x^2+9}\,dx = \frac{1}{2}\int \frac{2x}{x^2+9}\,dx = \frac{1}{2}\ln\left(x^2+9\right) + c$$

$t = x^2+9$

$dt = 2x\,dx$

$$= \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln t$$

$$\int \frac{1}{x^2+\underset{1}{9}}\,dx = \int \frac{1}{9\left(\frac{x^2}{9}+1\right)}\,dx = \frac{1}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx =$$

$\left(s = \frac{x}{3} \quad ds = \frac{1}{3}dx \right.$

$\left. 3ds = dx \right)$

$$= \frac{1}{9}\,3\,\mathrm{arctg}\left(\frac{x}{3}\right) + c = \frac{1}{3}\,\mathrm{arctg}\left(\frac{x}{3}\right) + c$$

3) $$\frac{0x^2+x-4}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)^2} + \frac{C}{x-2} = \frac{A\overset{(x-2)^2}{(x^2-4x+4)} + B(x-3) + C\overset{(x-2)(x-3)}{(x^2-5x+6)}}{(x-3)(x-2)^2} =$$

$$= \frac{\overset{0}{(A+C)}x^2 + \overset{1}{(-4A+B-5C)}x + \overset{-4}{4A-3B+6C}}{(x-3)(x-2)^2} = \frac{-1}{x-3} + \frac{2}{(x-2)^2} + \frac{1}{x-2}$$

$$\begin{aligned}
&\text{u } x^2: A + C = 0 \quad \longrightarrow \quad A + C = 0 \ /\cdot 8 \\
&\text{u } x: -4A + B - 5C = 1 \ /\cdot 3 \\
&\text{u } x^0: 4A - 3B + 6C = -4
\end{aligned} \Bigg\} \ -8A - 9C = -1$$

$$-C = -1 \rightarrow \underline{C = 1} \qquad \underline{B = 1 + 4A + 5C = 2}$$

$$\underline{A = -1}$$

$$\int \frac{x-4}{(x-3)(x-2)^2}\,dx = \int \left(\frac{-1}{x-3} + \frac{1}{x-2} + \frac{2}{\underset{\substack{s=x-2 \\ ds=dx}}{(x-2)^2}} \right) dx = -\ln|x-3| + \ln|x-2| + 2\frac{(x-2)^{-1}}{-1} + C$$

$(x-2)^{-2}$

$x \neq 2, 3$

$$\frac{3x^2+7x+7}{(x+1)(x^2+2x+2)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+2} = \frac{A(x^2+2x+2)+(Bx+C)(x+1)}{(x+1)(x^2+2x+2)} =$$

$$= \frac{A(\cancel{2x}\, x^2+2x+2) + Bx^2+Bx+Cx+C}{(x+1)(x^2+2x+2)} = \frac{3}{x+1} + \frac{1}{x^2+2x+2}$$

u x^2: $3 = A + B$

u x : $7 = 2A + B + C \quad (-1)$

u x^0: $7 = 2A + C$

$\Big\}\ 0 = -B \rightarrow B = 0$

$A = 3$

$C = 1$

$$\int\left(\frac{3}{x+1} + \frac{1}{x^2+2x+2}\right)dx = 3\ln|x+1| + \int\frac{1}{x^2+2x+1+1}\,dx =$$

$$= 3\ln|x+1| + \int\frac{1}{(x+1)^2+1}\,dx = 3\ln|x+1| + \operatorname{arctg}(x+1) + c$$

5)

$$\frac{4x^2 - x + 26}{(x-2)(x^2+16)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+16} = \frac{A(x^2+16)+(Bx+C)(x-2)}{(x-2)(x^2+16)} =$$

$$= \frac{Ax^2+16A+Bx^2-2Bx+Cx-2C}{(x-2)(x^2+16)} = \frac{2}{x-2} + \frac{2x+3}{x^2+16}$$

$$\left.\begin{array}{ll} u\,x^2: & A+B=4 \quad /\cdot(2) \\ u\,x: & -2B+C=-1 \end{array}\right\} \left.\begin{array}{l} 2A+C=7 \quad /\cdot 2 \\ 16A-2C=26 \end{array}\right\} 20A=40 \rightarrow \boxed{\begin{array}{l} A=2 \\ C=3 \\ B=2 \end{array}}$$

$$u\,x^0: 16A-2C=26$$

$$\int \frac{4x^2-x+26}{(x-2)(x^2+16)}\,dx = \int\left(\frac{2}{x-2} + \frac{2x+3}{x^2+16}\right)dx = 2\ln|x-2| + \underbrace{\ln(x^2+16) + \frac{3}{4}\operatorname{arctg}\left(\frac{x}{4}\right)}_{*} + \ldots$$

$$x \neq 2$$

$$* \int \frac{2x+3}{x^2+16}\,dx = \int \frac{2x}{x^2+16}\,dx + \int \frac{3}{x^2+16}\,dx = \ln(x^2+16) + 3\int \frac{1}{16\left(\frac{x^2}{16}+1\right)}\,dx = \; *$$

$$* = \ln(x^2+16) + \frac{3}{16}\int \frac{1}{\left(\frac{x}{4}\right)^2+1}\,dx = \underbrace{\ln(x^2+16) + \frac{3}{16}4\arctan\left(\frac{x}{4}\right) + C}_{*}$$

$$-\frac{x^3+3x^2+2x+3}{(x-2)(x+1)^3} = \frac{A}{x-2} + \frac{B}{(x+1)^3} + \frac{C}{(x+1)^2} + \frac{D}{x+1} =$$

$$= \frac{A(x^3+3x^2+3x+1)+B(x-2)+C(x+1)(x-2)+D(x-2)(x^2+2x+1)}{(x-2)(x+1)^3} =$$

$$= \frac{A(x^3+3x^2+3x+1)+B(x-2)+C(x^2-x-2)+D(x^3+\cancel{2x^2}+x-\cancel{2x^2}-4x-2)}{(x-2)(x+1)^3} =$$

$u\,x^3: A + D = -1$

$u\,x^2: 3A + C = -3$

$u\,x: 3A + B - C - 3D = -2$

$u\,x^0: A - 2B - 2C - 2D = -3$

$$\boxed{\begin{aligned} B &= 1 \\ A &= -1 \\ C &= D = 0 \end{aligned}}$$

$$= \frac{-1}{x-2} + \frac{1}{(x+1)^3}$$

$$\int \left(-\frac{1}{x-2} + \frac{1}{(x+1)^3}\right) dx = -\ln|x-2| + \frac{(x+1)^{-2}}{-2} + c$$

$x \neq 2, -1,\ c \in \mathbb{R}$

$$\frac{2x^4+4x^2+x+1}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)^2} + \frac{Dx+E}{x^2+1} =$$

$$= \frac{2}{x-1} + \frac{1}{(x^2+1)^2}$$