Inverzní matice

Hledáme A^{-1} pro $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}$

$$\begin{array}{cc} A & E \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] (-1) \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -2 & -1 & -1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right] (-2) \sim$$

$$(-3) \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -2 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & -3 & 6 \\ 0 & -2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & 4 \\ 0 & -2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right] :(-2) \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & 4 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right]$$

$$\begin{array}{cc} E & A^{-1} \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & -1 & 4 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

Zk. $A \cdot A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -1 & 4 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hledáme vl. čísla matice $B = \begin{bmatrix} 6 & -3 & 4 \\ -3 & 5 & -3 \\ -4 & 3 & -2 \end{bmatrix}$.

Matice má vl. číslo $\lambda = 5$

1) vl. čísla charakteristická rovnice: $\det(B - \lambda E) = 0$

$$0 = \det \begin{bmatrix} 6-\lambda & -3 & 4 \\ -3 & 5-\lambda & -3 \\ -4 & 3 & -2-\lambda \end{bmatrix} = (6-\lambda)(5-\lambda)(-2-\lambda) - 36 - 36 -$$

$$\begin{matrix} 6-\lambda & -3 & 4 \\ -3 & 5-\lambda & -3 \end{matrix}$$

$$- \left[-16(5-\lambda) - 9(6-\lambda) + 9(-2-\lambda) \right] =$$

$$= 0$$

$$(6-\lambda)(5-\lambda)(-2-\lambda)-72-(-80+16\lambda-54+\cancel{9\lambda}-18-\cancel{9\lambda})=$$

$$=(6-\lambda)(5-\lambda)(-2-\lambda)+80-16\lambda=(6-\lambda)\underline{(5-\lambda)}(-2-\lambda)+16\underline{(5-\lambda)}=0$$

$$(5-\lambda)\left[(6-\lambda)(-2-\lambda)+16\right]=0$$

$$(5-\lambda)\cdot(\lambda^2-4\lambda+4)=0$$

$$(5-\lambda)(\lambda-2)^2=0$$

$$\boxed{\lambda_1=5 \qquad \lambda_{2,3}=2}$$

Vlastní vektor příslušný k $\boxed{\lambda_1 = 5}$:

$$(B - \lambda E)\cdot \vec{x} = \vec{o} \quad , \quad (B - 5E)\cdot \vec{x} = \vec{o}$$

$$\begin{bmatrix} 1 & -3 & 4 \\ -3 & 0 & -3 \\ -4 & 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 \\ -3 & 0 & -3 \\ -4 & 3 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 4 \\ 0 & -9 & 9 \\ \cancel{0} & \cancel{-9} & \cancel{9} \end{bmatrix}$$

$$\begin{aligned} x - 3y + 4z &= 0 \\ -y + z &= 0 \end{aligned}$$

$z = t \quad (t \in R)$

$y = t$

$x = 3t - 4t = -t$

$$\vec{x_1} = \begin{bmatrix} -t \\ t \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad , \quad \boxed{\vec{x_1} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}$$

Zk :
$$\underline{\begin{bmatrix} 6 & -3 & 4 \\ -3 & 5 & -3 \\ -4 & 3 & -2 \end{bmatrix}} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix} = \underline{5} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{A \cdot \vec{x} = \lambda \vec{x}}$$

Vlastní vektor příslušný k $\lambda = 2$:

$x^2 - 4 = 0$

$\cancel{x = 3}$

$$(B - 2E)\cdot \vec{x} = \vec{o}$$

$$\begin{bmatrix} 4 & -3 & 4 \\ -3 & 3 & -3 \\ -4 & 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 & 4 \\ -3 & 3 & -3 \\ -4 & 3 & -4 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ \\ \end{matrix} \sim \begin{bmatrix} 4 & -3 & 4 \\ 0 & 3 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

$$4x - 3y + 4z = 0$$

$$3y = 0 \rightarrow y = 0$$

$$z = s,\ s \in \mathbb{R}$$

$$x = -s$$

$$\vec{x}_2 = \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix} = s \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \rightarrow \vec{x}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 & 4 \\ -3 & 5 & -3 \\ -4 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = 2 \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda^3 - 4\lambda^2 + 2\lambda + 1 = 0 \quad , \quad \lambda = \boxed{1}$$

$$\lambda^3 - 4\lambda^2 + 2\lambda + 1 = (\lambda - \boxed{1}) \cdot \underline{\underline{\text{NĚCO}}} = (\lambda - 1)(\lambda^2 - 3\lambda - 1)$$

$$(\lambda^3 - 4\lambda^2 + 2\lambda + 1) : (\lambda - 1) = \lambda^2 - 3\lambda - 1$$

$$\begin{array}{l}
-(\lambda^3 - \lambda^2) \\
\hline
\quad -3\lambda^2 + 2\lambda + 1 \\
-(-3\lambda^2 + 3\lambda) \\
\hline
\qquad -\lambda + 1 \\
\quad -(-\lambda + 1) \\
\hline
\qquad 0
\end{array}$$

$$(\lambda - 1)(\lambda^2 - 3\lambda - 1) = 0$$

$$\boxed{\lambda = 1}$$

$$\lambda^2 - 3\lambda - 1 = 0$$

$$\lambda_{2,3} = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\lambda_{2,3} = \frac{3 \pm \sqrt{13}}{2}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ \boxed{1} & 0 & 1 & \boxed{0} \\ 2 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{vmatrix} = \boxed{1} \cdot (-1)^{2+1} \overbrace{\begin{vmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix}}^{=0} + \cancel{0 \, (-1)^{2+2} \begin{vmatrix} 1 & 3 & 4 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$+ \, 1 \, (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 2 & 4 \\ 2 & 2 & 4 \end{matrix} + \cancel{\boxed{0} \, (-1)^{2+4} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}} =$$

$$= -\left(2 + \cancel{8} + \cancel{8} - \cancel{8} - \cancel{4} - \cancel{4}\right) = \underline{\underline{-2}}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 \\ 2 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{vmatrix} \begin{matrix} (-1) & (-2) & (-1) \end{matrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -2 & -4 \\ 0 & -2 & -3 & -4 \\ 0 & -1 & -2 & -3 \end{vmatrix} \begin{matrix} \\ (-1) \\ \\ (-2)!! \end{matrix} =$$

$$= \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -2 & -4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & +2 & 2 \end{vmatrix} \begin{matrix} \cdot\left(-\frac{1}{2}\right) \\ \\ 2 \\ \\ \end{matrix} = -\frac{1}{2} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -2 & -4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} = -\frac{1}{2} \cdot 1 \cdot (-2) \cdot (-1) \cdot 2 =$$

$$= -2$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 \\ 2 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{vmatrix} (-1) = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 1 \cdot (-1)^{3+1} \cdot \begin{vmatrix} 2 & 3 & 4 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= 1 \cdot (-1)^{2+2} \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = \underline{\underline{-2}}$$