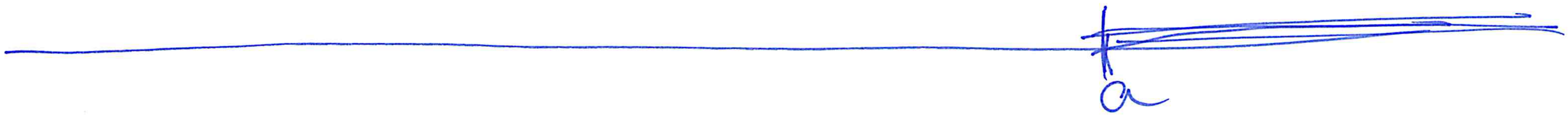
LIMITY V NEVLASTNÍCH BODECH

1) $f(x) = \dfrac{1-2x^3}{x^3+x^2+2}$ $\quad x \to +\infty$, $\quad x \to -\infty$

2) $f(x) = \dfrac{x+\sqrt{x^2+3}}{3x-1}$ $\quad x \to +\infty$, $\quad x \to -\infty$

3) $f(x) = \dfrac{1}{x+\sqrt{x^2+4x+1}}$ $\quad x \to +\infty$, $\quad x \to -\infty$

OKOLÍ NEVLASTNÍCH BODŮ



$$\mathcal{U}_a(+\infty) = (a, +\infty)$$

$$\mathcal{U}_a(-\infty) = (-\infty, a)$$

$$\Big(\underset{x \in \mathbb{R}}{\mathcal{U}_\varepsilon(x)} = (x-\varepsilon, x+\varepsilon) \Big)$$

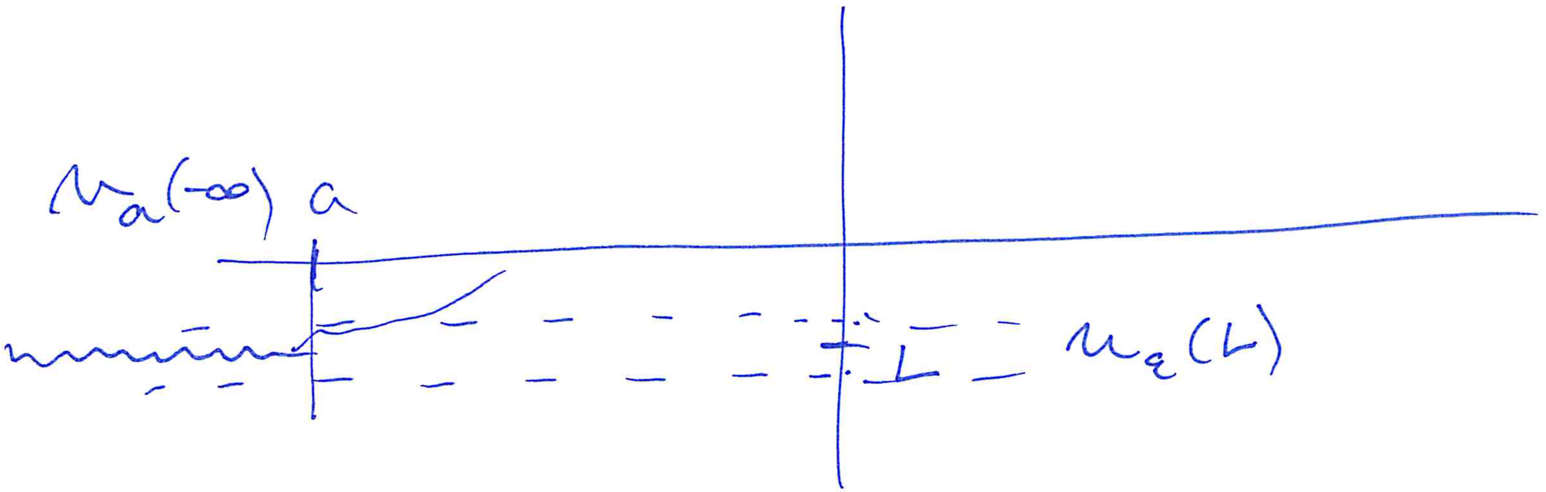
DEFINICE LIMIT V NEVLASTNÍCH BODECH

$$\cancel{(\forall a \in R)(\exists \delta > 0)(\forall x \in P_\delta(x_0))(f(x) \in U_a(L))}$$

$x_0 \in \{+\infty, -\infty\}$

Pak řekneme, že funkce f má v bodě x_0 limitu rovnu $L \in \mathbb{R}$ (L je konečné), pokud

$$(\forall \varepsilon > 0)(\exists a \in R)(\forall x \in U_a(x_0))(f(x) \in \underbrace{U_\varepsilon(L)}_{(L-\varepsilon,\, L+\varepsilon)}$$



1) $$f(x) = \frac{1-2x^3}{x^3+x^2+2} = \frac{\cancel{x^3}\left(\frac{1}{x^3}-2\right)}{\cancel{x^3}\left(1+\frac{1}{x}+\frac{2}{x^3}\right)} = \frac{\frac{1}{x^3}-2}{1+\frac{1}{x}+\frac{2}{x^3}}$$

co použijeme: $\frac{1}{x} \to 0$ pro $x \to +\infty$
a $x \to -\infty$

věty o limitách a aritmetických operacích:

$$\frac{1}{x^3} - 2 = \underbrace{\frac{1}{x}\cdot\frac{1}{x}\cdot\frac{1}{x}}_{\to 0\cdot 0\cdot 0} - 2 \to -2$$

věta o limitě součinu, součtu, rozdílu,
podílu

2) $f(x) = \dfrac{x+\sqrt{x^2+3}}{3x-1} = \dfrac{\cancel{x}\left(1 \pm \sqrt{1+\frac{3}{x^2}}\right)}{\cancel{x}\left(3-\frac{1}{x}\right)}$ $\quad x \to -\infty$

$$\sqrt{x^2+3} = \sqrt{x^2\left(1+\frac{3}{x^2}\right)} = \sqrt{x^2}\sqrt{1+\frac{3}{x^2}}$$

$$= |x| = -x$$

$$\sqrt{x^2+3} = |x|\sqrt{1+\frac{3}{x^2}} \overset{x<0}{=} -x\sqrt{1+\frac{3}{x^2}}$$

navíc použijeme větu o limitě odmocniny a limitě složené funkce

$$\frac{1\pm\sqrt{1+\frac{3}{x^2}}}{3-\frac{1}{x}} \to \frac{1\pm\sqrt{1+0}}{3-0} = \frac{2}{3} \quad 0$$

3) $x \to +\infty$

$$\frac{1}{x+\sqrt{x^2+4x+1}} = \frac{1}{x\left(1+\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}\right)} =$$

$$= \underbrace{\frac{1}{x}}_{\to 0} \cdot \underbrace{\frac{1}{1+\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}}}_{\to \frac{1}{2}} \to 0 \cdot \frac{1}{2} = 0$$

$x \to -\infty \qquad |x| = -x \quad (x<0)$

$$\sqrt{x^2+4x+1} = \sqrt{x^2\left(1+\frac{4}{x}+\frac{1}{x^2}\right)} = |x|\sqrt{1+\frac{4}{x}+\frac{1}{x^2}} = -x\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}$$

$$\frac{1}{x+\sqrt{x^2+4x+1}} = \frac{1}{x\left(1-\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}\right)} \frac{\left(1+\sqrt{}\right)}{\left(1+\sqrt{}\right)} =$$

$$= \frac{\overbrace{1+\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}}^{\to 2}}{x\left(1-\left(1+\frac{4}{x}+\frac{1}{x^2}\right)\right)} = \frac{1+\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}}{x\left(-\frac{4}{x}-\frac{1}{x^2}\right)} =$$

$$= \frac{1+\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}}{-4-\frac{1}{x}} \to \frac{1+\sqrt{1}}{-4-0} = -\frac{1}{2}$$

Příště: sepsat použité věty