$\frac{1}{x}$ $\frac{1}{x^2}$ $\operatorname{sgn}(x)$

$x \to 0$

$x \to 0^+$

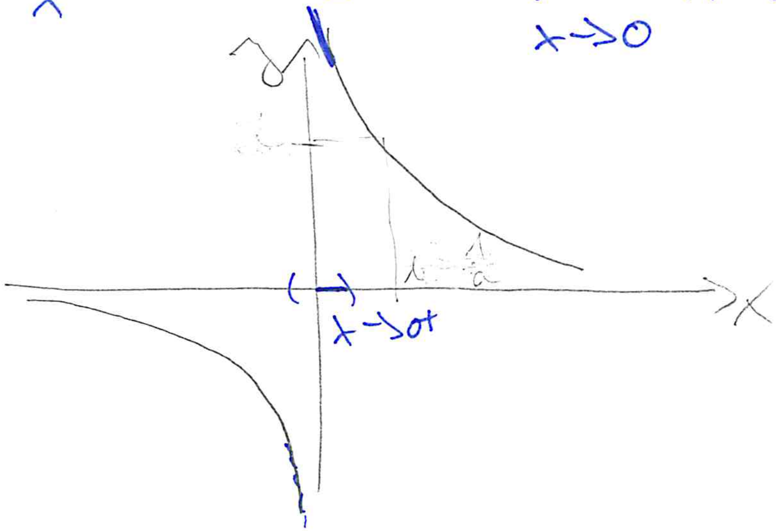
$x \to 0^-$

značení:

$\lim\limits_{x \to 0^+} \frac{1}{x} = +\infty$ "$\frac{1}{0^+}$" $= +\infty$

$\lim\limits_{x \to 0^-} \frac{1}{x} = -\infty$ $\lim\limits_{x \to 0} \frac{1}{x}$ neexistuje

graf:

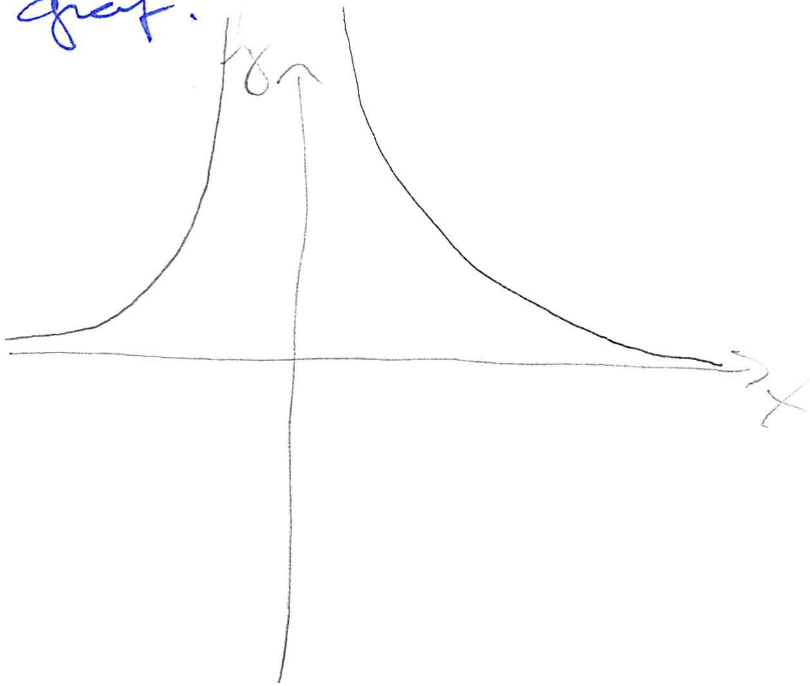


$$\lim_{x \to 0^+} \frac{1}{x^2} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x^2} = +\infty$$

$$\lim_{x \to 0} \frac{1}{x^2} = +\infty$$

graf:

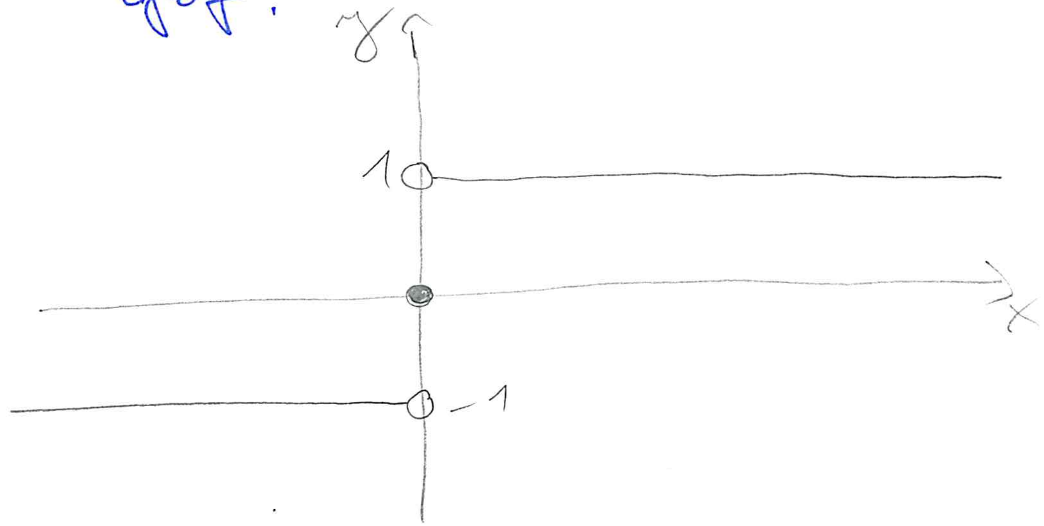


$$\lim_{x \to 0^+} \operatorname{sgn}(x) = 1$$

$$\lim_{x \to 0^-} \operatorname{sgn}(x) = -1$$

$\lim_{x \to 0} \operatorname{sgn}(x)$ neexistuje

graf:



Má-li funkce v bodě x_0 jednostranné různé vlastní limity, pak říkáme, že funkce má v bodě x_0 nespojitost typu skoku.

$$\frac{x^2+3x+2}{x^2-1}$$

$x \to 1$?

$x \to -1$?

$$\lim_{x \to 1^+} \frac{\overbrace{x^2+3x+2}^{=6>0}}{x^2-1} = +\infty$$

$$\underbrace{(x-1)}_{0^+}\underbrace{(x+1)}_{=2>0}$$

$$\lim_{x \to 1^-} \frac{x^2+3x+2}{x^2-1} = -\infty$$

oboustranná limita neexistuje

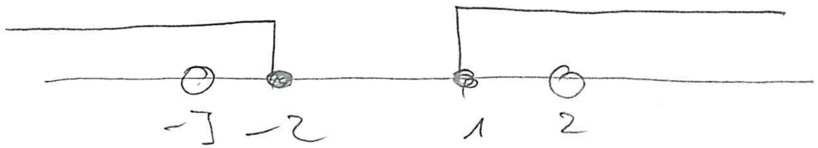
$x \to -1$

oboustranná i jednostranné limity rovny $-\frac{1}{2}$

(viz začátek sešitu)

$$\frac{x}{2-\sqrt{x^2+x-2}}$$

$$x^2+x-2 \ge 0$$



$$x \in (-\infty, -2] \cup [1, +\infty)$$

$$\frac{x\left(2+\sqrt{x^2+x-2}\right)}{6-x-x^2}$$

$x \to -3: \quad -3 \cdot 4 = -12$

$$\text{"}\frac{-12}{0}\text{"}$$

$$2-\sqrt{x^2+x-2} = 0 \qquad x_1 = -3, \quad x = 2$$

$x \to -3 \qquad \text{"}\frac{-3}{0}\text{"}$

$x \to 2 \qquad \text{"}\frac{2}{0}\text{"}$

$$6-x-x^2 = (2-x)(x+3)$$

$$\frac{x\left(2+\sqrt{x^2+x-2}\right)}{(2-x)(x+3)}$$

$x=-3 \quad -\frac{12}{5}$

$$\boxed{\lim_{x\to-3^+} \frac{x}{2-\sqrt{x^2+x-2}} = \cancel{+\infty}\ -\infty}$$

$$\boxed{\lim_{x\to-3^-} \frac{x}{2-\sqrt{x^2+x-2}} = +\infty}$$

Oboustranná limita neexistuje

$\sqrt{x^2+x+1} - x$ $\qquad x \to +\infty$

$\qquad x \to -\infty$

$x \to +\infty$ $\qquad$ "$+\infty - (+\infty)$" $\qquad$ bude upravovat

$x \to -\infty$

$$x^2+x+1 = \underbrace{x^2}_{\to +\infty}\underbrace{\left(1+\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}\right)}_{\to 1} \to +\infty$$

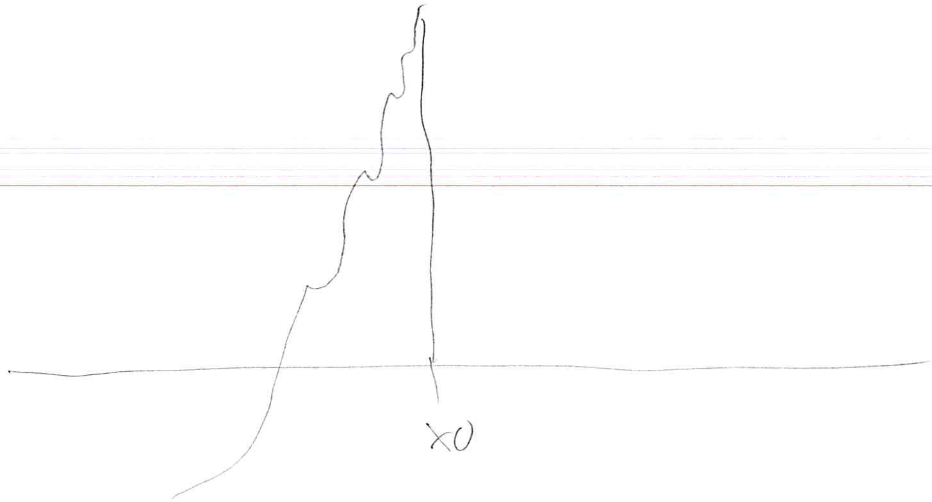
$$\lim_{x \to -\infty} \sqrt{x^2+x+1} - x = +\infty$$

$$\lim_{x \to +\infty} \sqrt{x^2+x+1} - x = \frac{1}{2}$$

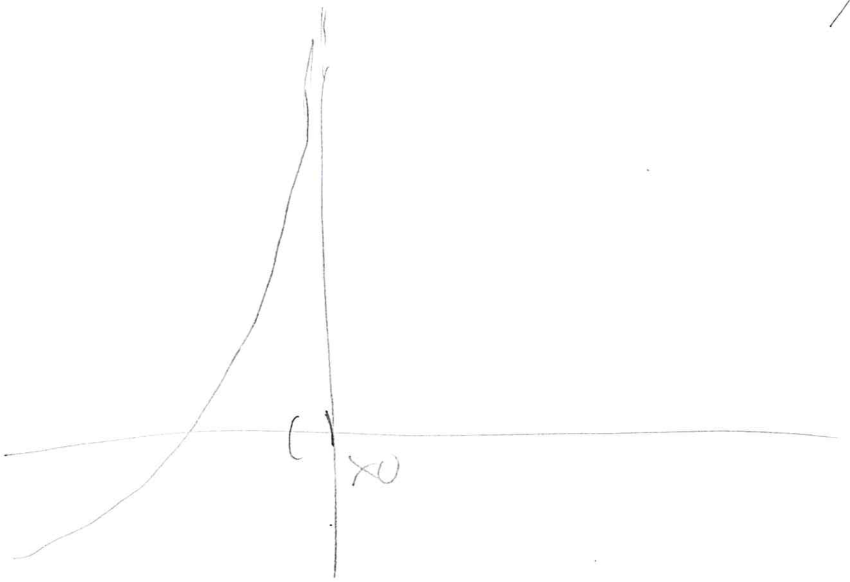
$$\frac{\left(\sqrt{x^2+x+1} - x\right)}{1} \frac{\left(\sqrt{x^2+x+1} + x\right)}{\left(\sqrt{x^2+x+1} + x\right)} = \frac{x+1}{\sqrt{x^2+x+1} + x} =$$

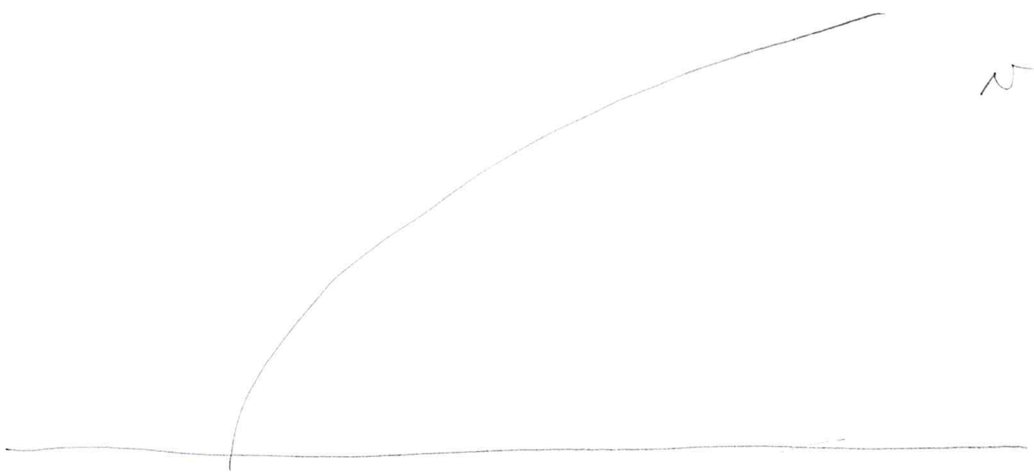
$$= \frac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + 1\right)} \longrightarrow \frac{1}{2} \quad \text{for } x \to +\infty$$

Nevlastní limity - definice



ve vlastním bodě





v nevlastním bodě

okolí nekonečna

$$\mathcal{U}_a(+\infty) = (a, +\infty)$$

$$\mathcal{U}_a(-\infty) = (-\infty, a)$$

$$\forall \mathcal{U}(L)\,\exists \cancel{\mathcal{U}}\mathcal{P}(x_0)\,(\forall x \in \mathcal{U}(x_0))\,(f(x) \in \mathcal{U}(L))$$

$$\lim_{x \to x_0} f(x) = L$$

1) $x_0 \in \mathbb{R}$, $L \in \mathbb{R}$

$$(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x \in \cancel{U_\delta(x_0)}\ P_\delta(x_0))(f(x) \in U_\varepsilon(\cancel{f(x_0)}\ L))$$

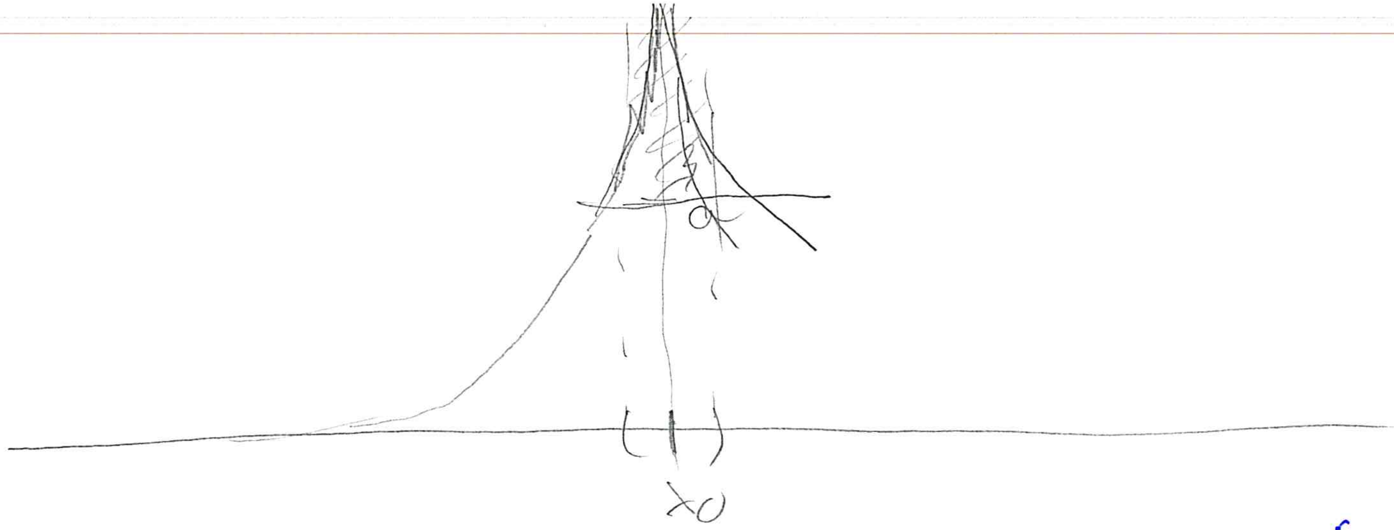
Obecně:

$$(\forall U(L))(\exists \cancel{U}\ P(x_0))(\forall x \in P(x_0))(f(x) \in U(L))$$

2) $x_0 \in \mathbb{R}$, $L = +\infty$

$$(\forall a \in \mathbb{R})(\exists \delta > 0)(\forall x \in P(x_0))(f(x) \in (a, +\infty))$$

$L = -\infty$ — " — $(-\infty, a)$



$(x_0-\delta, x_0) \cup (x_0, x_0+\delta)$

Jednostranné nevlastní limity – místo prstencového okolí uvažujeme pravé nebo levé okolí

$(x_0, x_0+\delta)$ $\qquad$ $(x_0-\delta, x_0)$

$x \to x_0^-$: $(\forall a \in \mathbb{R})(\exists \delta > 0)(\forall x \in (x_0-\delta, x_0))(f(x) \in (a, +\infty))$

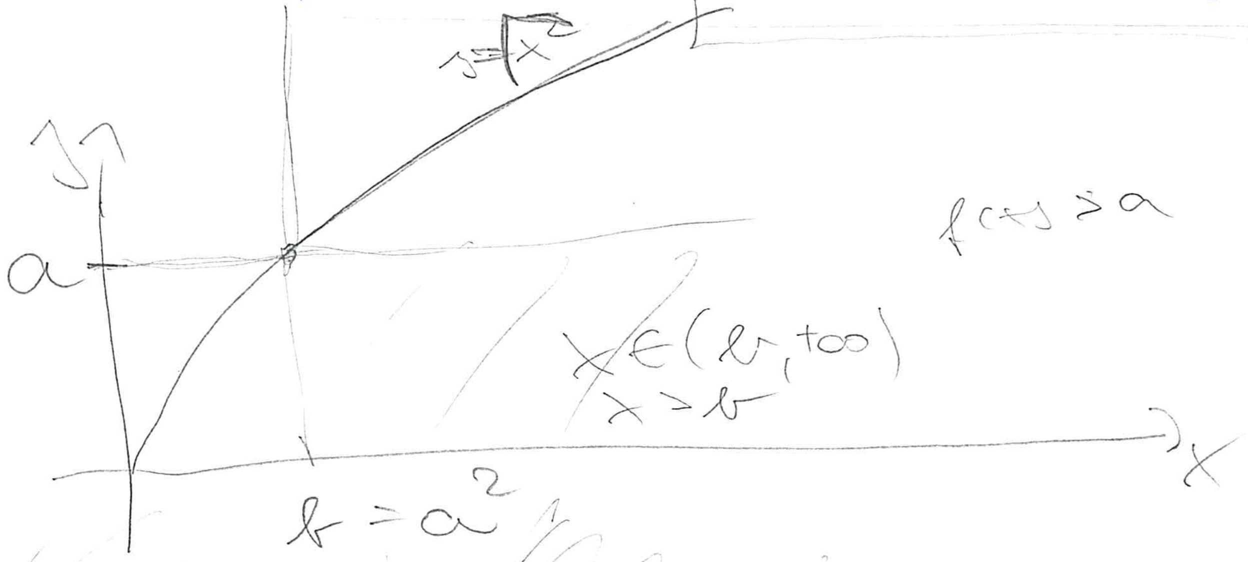
$x \to x_0^+$ $\qquad$ $(x_0, x_0+\delta)$

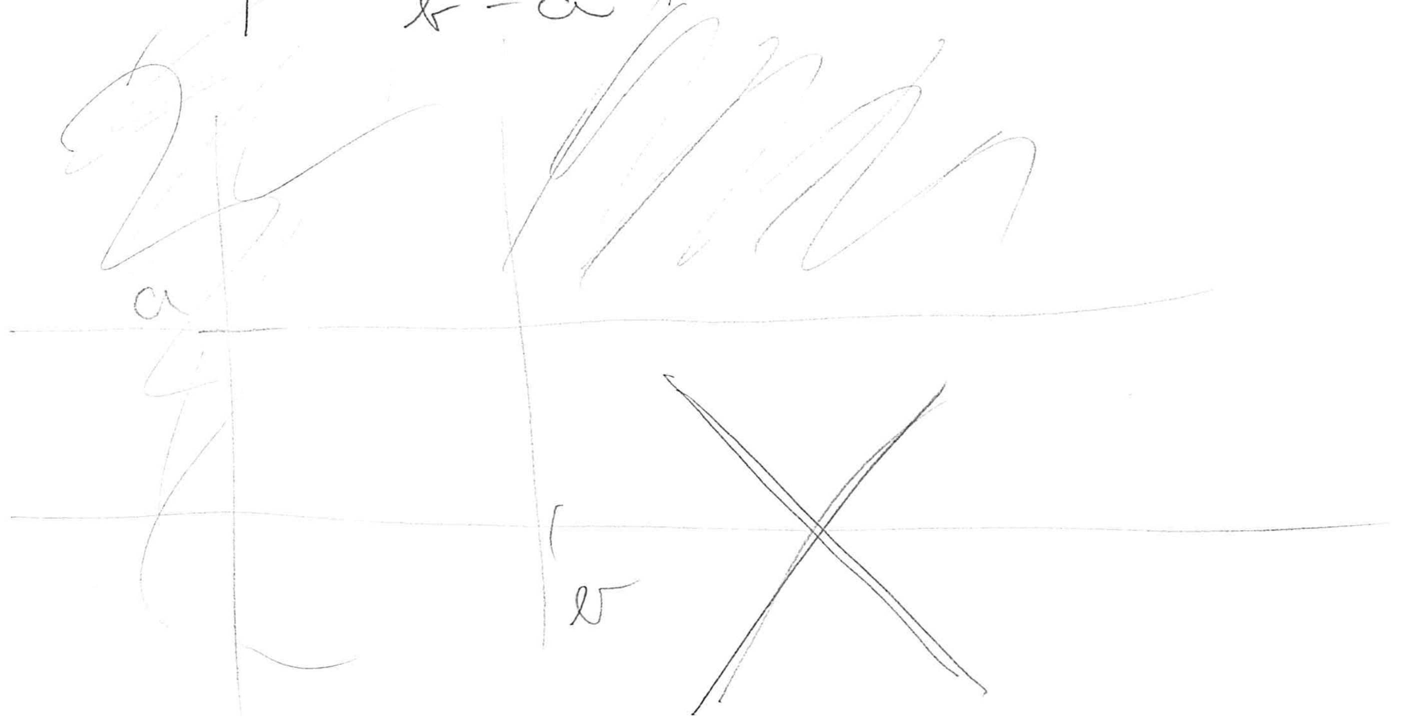
3) $x_0 = +\infty$, $L = +\infty$ další 3 případy: $-\infty$ $+\infty$

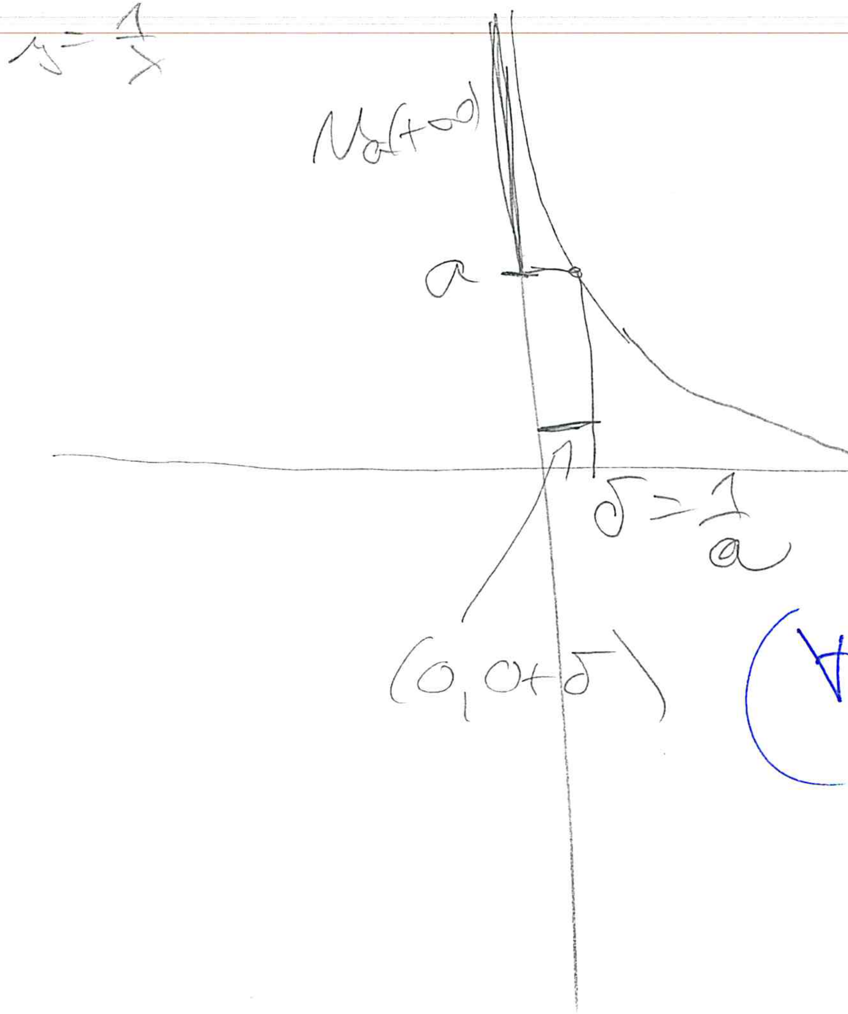
$+\infty$ $-\infty$

$-\infty$ $-\infty$

$(\forall a \in \mathbb{R})(\exists b \in \mathbb{R})\boxed{(\forall x \in (b,+\infty))(f(x) \in (a,+\infty))}$







$x_0 = 0$

$x \to 0^+$

$L = +\infty$

$(\forall x \in (0, \delta))(f(x) \in \cancel{(a, +\infty)})$

$(a, +\infty)$

$$\frac{1}{x^2} = \frac{1}{x} \cdot \frac{1}{x} \longrightarrow (+\infty)\cdot(+\infty) = +\infty \quad \text{pro } x \to 0^+$$

věta o limitě součinu

$x \to 0^-$ $$\frac{1}{x^2} = \frac{1}{x} \cdot \frac{1}{x} \to (-\infty)\cdot(-\infty) = +\infty$$

~~O~~perace s nekonečny – viz [JV], str. 64

značíme: „$\dfrac{x}{0^+}$“ je pro $x > 0$ $+\infty$

$x < 0$ $-\infty$

$x \neq 0$ $\operatorname{sgn}(x)\cdot(+\infty)$

„$\dfrac{x}{0^-}$“ pro $x \neq 0$ $\operatorname{sgn}(x)(-\infty)$

pro ~~$x \in P_\delta(-3)$ pro malé δ je~~ (leží mi na hranici ~~[illegible]~~ případně za def. obor i def. obor)

$$\frac{x}{2-\sqrt{x^2+x-2}} = \underbrace{\frac{x\left(2+\sqrt{x^2+x-2}\right)}{2-x}}_{\to -\frac{12}{5} \text{ pro } x \to -3} \cdot \frac{1}{x+3}$$

$\frac{1}{x+3} \to +\infty$ pro $x \to -3^+$

$\frac{1}{x+3} \to -\infty$ pro $x \to -3^-$

věta o limitě součinu:

$$-\frac{12}{5} \cdot (+\infty) = -\infty$$

$$-\frac{12}{5}(-\infty) = +\infty$$