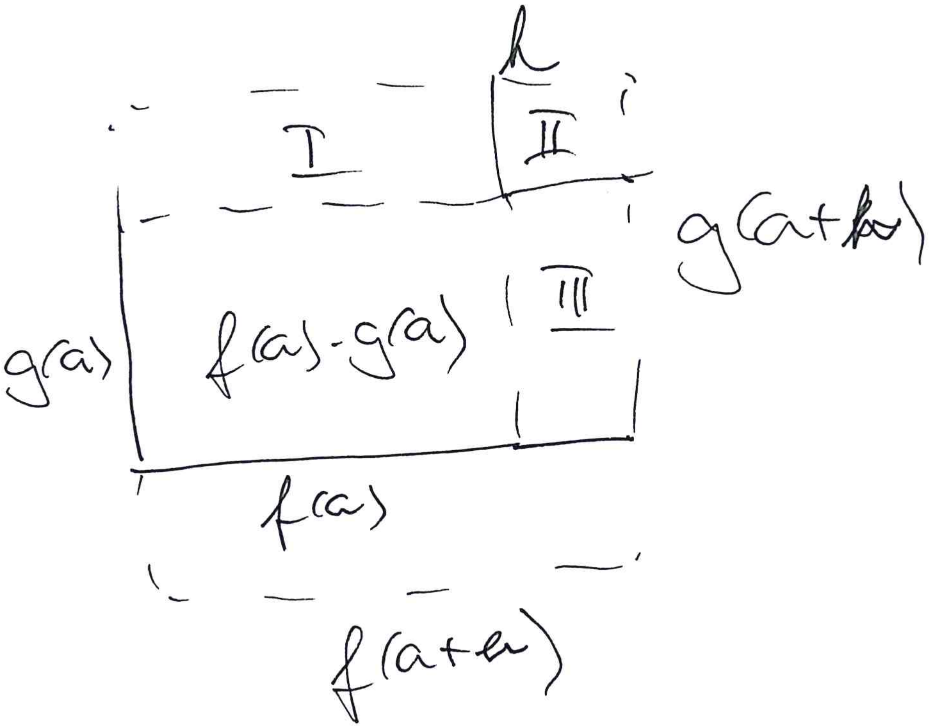
II 1

$$(fg)' \overset{?}{=} f(a)\cdot g'(a) + f'(a)\cdot g(a)$$

$$(f\cdot g)(a) = f(a)\cdot g(a)$$

$$(f\cdot g)(a+h) = f(a+h)\cdot g(a+h)$$

$$\underline{\underline{f(a+h)\cdot g(a+h) - f(a)g(a)}} = \mathrm{I} + \mathrm{II} + \mathrm{III}$$



$$\mathrm{I} = f(a)\,\big(g(a+h) - g(a)\big)$$

$$\mathrm{II} = \big(\underbrace{f(a+h) - f(a)}_{\Delta f}\big)\big(\underbrace{g(a+h) - g(a)}_{\Delta g}\big)$$

$$\mathrm{III} = \big(f(a+h) - f(a)\big)\cdot g(a)$$

$$\to \frac{\mathrm{I}}{h} + \frac{\mathrm{II}}{h} + \frac{\mathrm{III}}{h} = \underbrace{f(a)\frac{g(a+h)-g(a)}{h}}_{f(a)\cdot g'(a)} + \underbrace{\frac{\Delta f}{h}\cdot\frac{\Delta g}{h}\cdot h}_{f'(a)\cdot g'(a)\cdot 0} + f'(a)g$$

$h \to 0$:

$(x^n)' = ?$

$(x^1)' = (x)' = 1 \,,\; (x^0)' = 0$

$(x^2)' = (x \cdot x)' = (x)' \cdot x + x\,(x)' =$
$= 1 \cdot x + x \cdot 1 = 2x$

$(x^3)' = (x^2 \cdot x)' = (x^2)' \cdot x + x^2 (x)' =$
$= 2x \cdot x + x^2 \cdot 1 = 3x^2$

$(x^4)' = (x^3 \cdot x)' = \ldots = 4x^3$

$\vdots$

$(x^{15})' = (x^{14} \cdot x)'$

matematická indukce

$(x^{n+1})' = (x^n \cdot x)' =$

předpokládáme: $(x^n)' = n x^{n-1}$

$= (x^n)' \cdot x + x^n (x)' = n x^{n-1} \cdot x +$
$+ x^n \cdot 1 = n x^n + 1 \cdot x^n = (n+1)\, x^n$