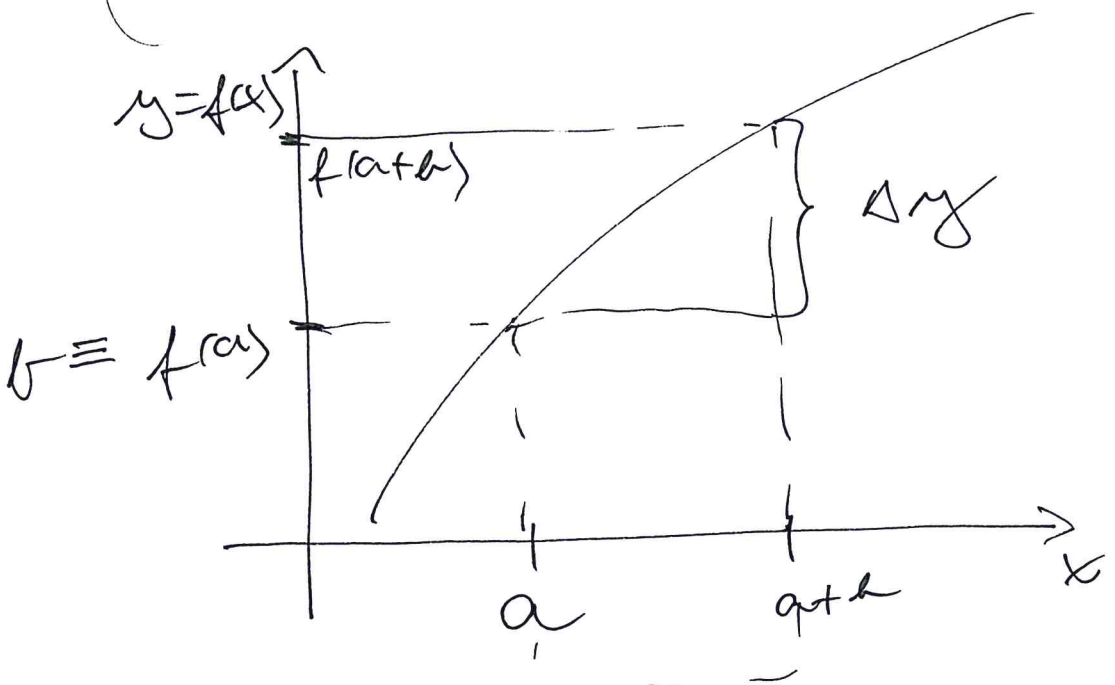
$(\sqrt[n]{x})' = ?$

$(f^{-1})' = ?$ ↑ $(x^n)'$



$h = \Delta x$

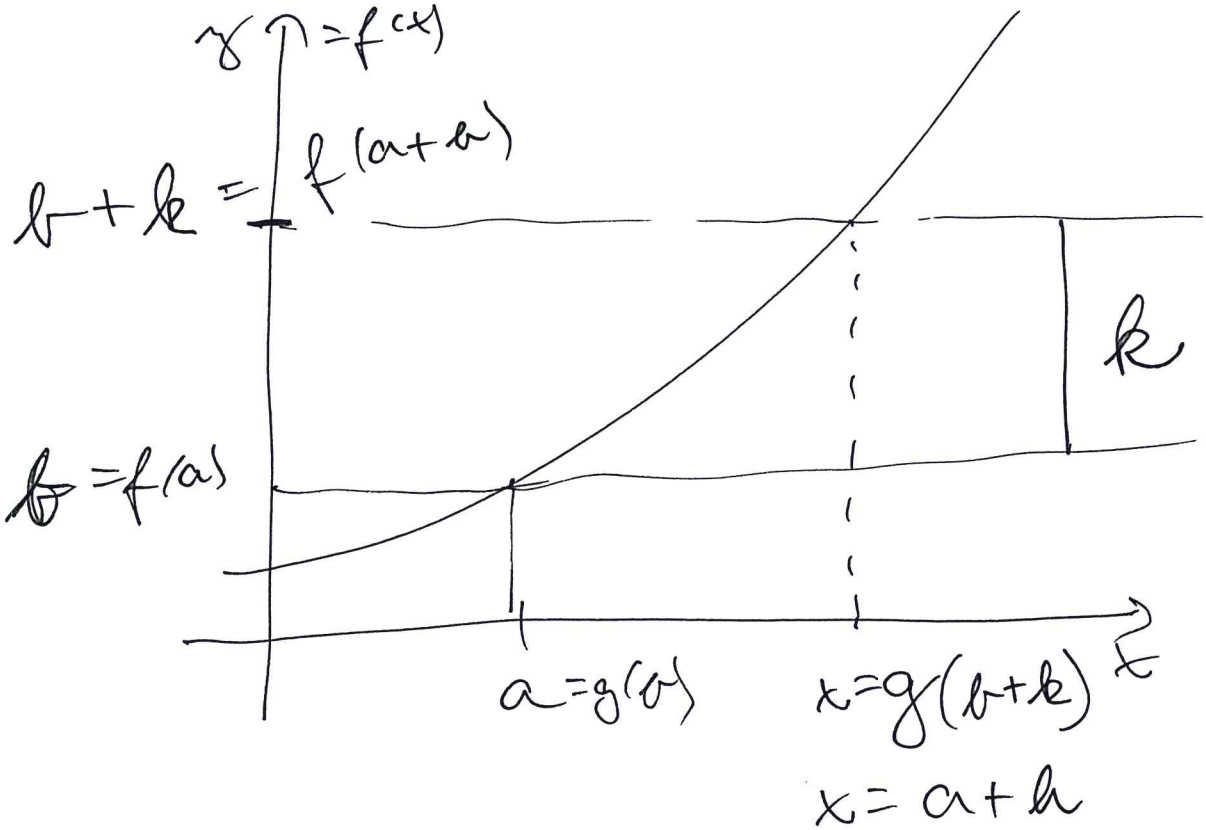
$b = f(x)$

$x = g(y)$

↑ inverzní funkce k f

$$f'(a) : \frac{\Delta y}{\Delta x}$$

$$g'(b) : \frac{\Delta x}{\Delta y} = \frac{1}{\frac{\Delta y}{\Delta x}} = \frac{1}{f'(a)}$$



$$g'(b) = \frac{g(b+k) - g(b)}{k} =$$

$$= \frac{a+h-a}{f(a+h)-f(a)} = \frac{h}{f(a+h)-f(a)} =$$

$$= \frac{1}{f'(a)}$$

Δείχνει:

$$g'(b) = \frac{1}{f'(g(b))}$$

$\left(\sqrt[n]{x}\right)' = ?$

III 3

$f(x) = x^n = y \qquad y>0,\ x>0$

$x = \sqrt[n]{y}$

$g(y) = \sqrt[n]{y}$

víme: $f'(x) = n x^{n-1}$

víme: $g'(y) = \dfrac{1}{f'(x)} = \dfrac{1}{n x^{n-1}} =$

$= \dfrac{1}{n\left(\sqrt[n]{y}\right)^{n-1}} =$

$= \dfrac{1}{n} \cdot y^{\frac{1-n}{n}} =$

$= \dfrac{1}{n} y^{\frac{1}{n}-1}$

závěr: $\left(\sqrt[n]{y}\right)' = \dfrac{1}{n} y^{\frac{1}{n}-1}$

$\left(y^{\frac{1}{n}}\right)' = \dfrac{1}{n} y^{\frac{1}{n}-1}$