

$$\left(\frac{1}{x}\right)' = ?$$

$$\left(f(g(x))\right)' = ?$$

$$\left(\frac{f}{g}\right)' = \left(f \cdot g^{-1}\right)' = ?$$

$$\left(\frac{1}{x}\right)': f(x) = \frac{1}{x}$$

$$f(a+h) - f(a) = \frac{1}{a+h} - \frac{1}{a}$$

$$\frac{f(a+h) - f(a)}{h} = \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \frac{\frac{a - (a+h)}{(a+h)a}}{h}$$

$$= \frac{-h}{(a+h) \cdot a} \cdot \frac{1}{h} = \frac{-1}{(a+h)a}$$

$$h=0: \frac{-1}{a^2}$$

$$\text{zürück: } \left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$\left( f(g(x)) \right)' = ? \quad \frac{dz}{dx}$$

$$y = g(x) \quad g'(x) = \frac{dy}{dx} = \frac{\Delta y}{\Delta x}$$

$$z = f(y) \quad f'(y) = \frac{dz}{dy} = \frac{\Delta z}{\Delta y}$$

$$g'(x) \cdot f'(y) = \frac{dy}{dx} \cdot \frac{dz}{dy} = \frac{dz}{dx}$$

Závěr:

$$\left( f(g(x)) \right)' = g'(x) \cdot f'(y)$$

$$\begin{aligned} \left( \frac{f}{g} \right)' &= \left( f \cdot g^{-1} \right)' = f' \cdot g^{-1} + f \cdot (g^{-1})' = \\ &= f' \cdot g^{-1} + f \cdot \frac{-1}{g^2} \cdot g' = \\ &= \frac{f'}{g} - \frac{fg'}{g^2} = \frac{f'g - fg'}{g^2} \end{aligned}$$

Závěr: 
$$\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}$$