$$\sum_{k=1}^{n} \frac{1}{k^3} \quad a_k$$

k^3	2^3
$\sqrt{k}$	$\sqrt{2}$
$\sqrt{k^3}$	$\sqrt{2^3}$
k	2
k^2	2^2

$$b_k = \frac{1}{\sqrt{k}}$$

$$c_k = \frac{1}{\sqrt{k^3}}$$

$$d_k = \frac{1}{k}$$

$$k^{\frac{1}{2}} < k^1 < k^{\frac{3}{2}} < k^2 < k^3$$

$$e_k = \frac{1}{k^2}$$

$$\frac{1}{k^3} \le \frac{1}{k^2} \le \frac{1}{\sqrt{k^3}} \le \frac{1}{k} \le \frac{1}{\sqrt{k}}$$

konv. diverg. $(s=+\infty)$

$$\sum_{k=1}^{+\infty} \frac{1}{k^3} < +\infty$$ konv.

$$\sum_{k=1}^{+\infty} \frac{1}{\sqrt{k}} = +\infty$$

Věta 3.2.2 str. 83 [JV]

$$\sum_{k=1}^{+\infty} \frac{1+\sqrt{k}}{2+k^3} = s$$

? $s = +\infty$ ✗

$s < +\infty$ ✓

$$\underbrace{\frac{1+\sqrt{k}}{2+k^3}}_{=b_k} = \underbrace{\frac{\sqrt{k}}{k^3}}_{=a_k} \cdot \frac{\frac{1}{\sqrt{k}}+1}{\frac{2}{k^3}+1} \to \frac{0+1}{0+1} = 1$$

pro $k \to +\infty$

$$\frac{\sqrt{k}}{k^3} = \frac{1}{k^{\frac{5}{2}}}$$

$$\sum_{k=1}^{+\infty} \frac{1}{k^{\frac{5}{2}}}$$

$$\frac{1}{k^{\frac{5}{2}}} \leq \frac{1}{k^2}$$

$$\sum \frac{1}{k^{5/2}} < +\infty \Leftarrow \sum \frac{1}{k^2} < +\infty$$

$\varepsilon = 0.1$ pro velké k je

$$0.9 < \frac{\frac{1}{\sqrt{k}}+1}{\frac{2}{k^3}+1} < 1.1$$

$$\leq \frac{1}{k^{5/2}} \cdot 1.1$$

geometrická posloupnost:

$$\frac{a_{k+1}}{a_k} = q \quad \text{.. nezávisí na } k$$

$$a_k = \frac{1}{2^k} \quad \frac{a_{k+1}}{a_k} = \frac{\frac{1}{2^{k+1}}}{\frac{1}{2^k}} = \frac{2^k}{2^{k+1}} = \frac{1}{2}$$

závisí $\sum \frac{k}{2^k} < +\infty$

$$a_k = \frac{k}{2^k} \quad \frac{a_{k+1}}{a_k} = \frac{\frac{k+1}{\cancel{2}2^{k}}}{\frac{k}{\cancel{2^k}}} = \frac{k+1}{2k}$$

závisí na k - není geometrická

$$\lim_{k\to+\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to+\infty} \frac{k+1}{2k} = \frac{1}{2}$$

z definice limity: pro velká k je

$$0.4 = \frac{1}{2} - 0.1 < \frac{k+1}{2k} < \frac{1}{2} + 0.1 = 0.6$$

$$\frac{a_{k+1}}{a_k} < 0.6 = q$$

$a_n < a_1 q^{n-1}$

$a_n < a_{k_0} q^{n-k_0}$

konv. geometrická $q = 0.6$ $\sum_{n=1}^{\infty} a_{k_0} q^{n-k_0}$

$$s_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}$$

$\frac{1}{k}$ je klesající posloupnost

$s_1 = \frac{1}{1}$

$s_2 = 1 - \frac{1}{2}$

$s_3 = 1 - \frac{1}{2} + \frac{1}{3}$

$s_4 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

$s_1 > s_2 < s_3 > s_4$

$s_1 > s_3 > s_5 > s_7$

$s_2 < s_4 < s_6 < s_8$

odtud plyne, že s_1, s_3, s_5 ... liché

s_2, s_4, s_6 ... nelich

rovnají se?

$$s_{2k+1} - s_{2k+2} = -a_{2k+2} \to 0$$

pro $k \to +\infty$

$\frac{1}{k} \to 0$ pro $k \to +\infty$

Leibnizovo kritérium str. 96 3.3.1 [NV]

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots$$

$$0 + \frac{1}{2} + 0 - \frac{1}{4} + 0 + \frac{1}{6} + 0 - \frac{1}{8} \cdots$$

$$1 + 0 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \cancel{0} + \frac{1}{7} - \frac{1}{4}$$

Absolutní konvergence - definice 3.1.14

str. 82 [JV]

$$\sum_{k=1}^{\infty} \frac{1}{k} = +\infty$$

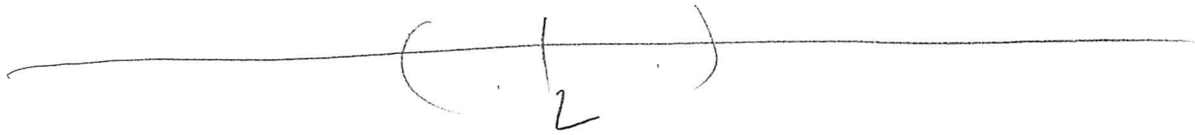
tedy $\sum \frac{(-1)^{k+1}}{k}$ není absolutně konvergentní

Věta 3.4.6 o přerovnání abs. konv. řady

Definice 2.4.6 str. 67

Cauchyovská posloupnost

$$a_n \in (L-\varepsilon, L+\varepsilon)$$



$$|a_n - L| < \varepsilon$$

Reálná čísla

$\sqrt{2} \notin \mathbb{Q}$

$$a_1 = 1.4$$
$$a_2 = 1.41$$
$$a_3 = 1.414$$
$$a_4 = 1.4142$$
$$a_5 = 1.41423$$
$$\vdots$$

$$\lim_{n \to +\infty} a_n = \sqrt{2}$$

$$|x+y| \leq |x| + |y|$$

$$|x+y+z| = |(x+y)+z| \leq |x+y| + |z| \leq$$

$$\leq |x| + |y| + |z|$$

$$s_m - s_n =$$

$$s_{10} - s_7 = \underline{\cancel{a_1} + \dots + a_{10}} - (\cancel{a_1} + \dots a_7)$$

$$= a_8 + a_9 + a_{10}$$

$$|s_m - s_n| = |S_m - S_n|$$

$$S_m = |a_1| + |a_2| + \dots |a_m|$$