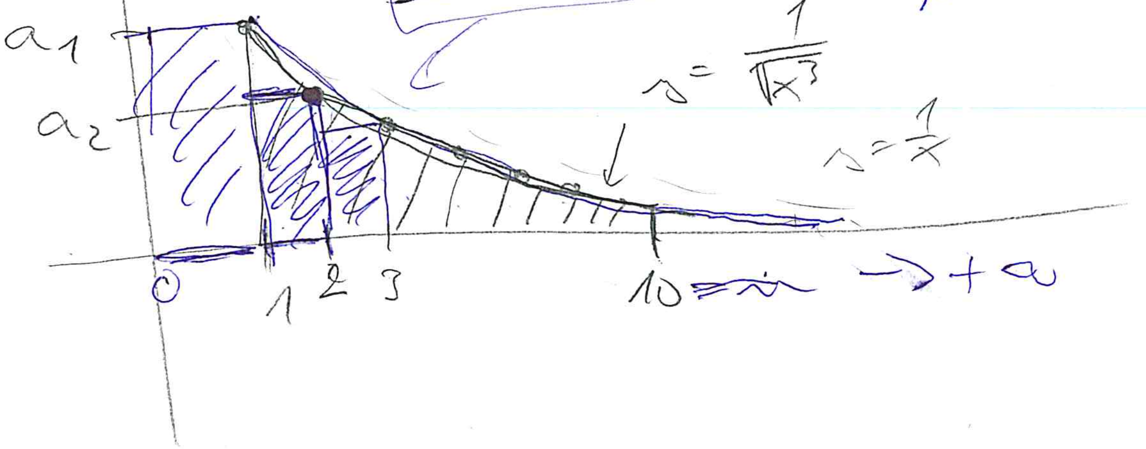
$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3}} \qquad \sum_{k=1}^{10} \frac{1}{\sqrt{k^3}}$$

$y = \frac{1}{\sqrt{x^3}}$ klesající na $[1,+\infty)$

$$\boxed{a_2 + \dots + a_{10} \leq \int_1^{10} \frac{1}{\sqrt{x^3}}\,dx}$$



$$\int_1^{\infty} \frac{1}{\sqrt{x^3}}\,dx = \int_1^{+\infty} x^{-\frac{3}{2}}\,dx = \left[-2x^{-\frac{3}{2}+1}\right]_1^{+\infty} =$$

$-\frac{3}{2}+1 = -\frac{1}{2}$

$$= \lim_{x\to+\infty} -2x^{-\frac{1}{2}} - \left(-2 \cdot 1^{-\frac{1}{2}}\right)$$

$$\frac{-2}{\sqrt{x}}$$

$$0 \quad + 2 \quad = 2$$

Integrální kritérium:

Je-li f klesající na $[1,+\infty)$, pak $\sum_{k=1}^{\infty} f(k)$ je konečná $\Leftrightarrow$ (N) $\int_1^{+\infty} f(x)\,dx$ je konečný

$$T_n(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} =$$

$0! = 1$

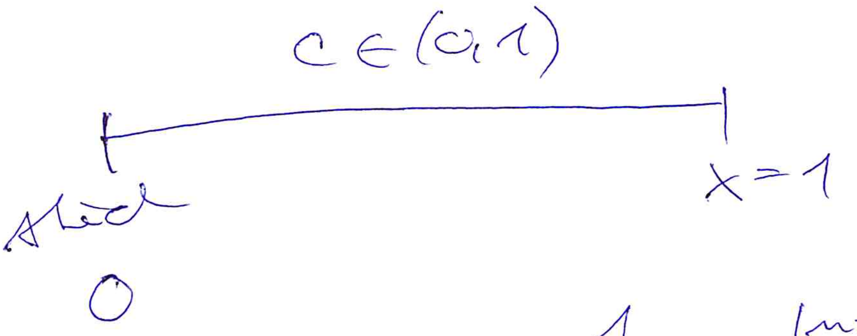
$$= \sum_{k=0}^{n} \frac{x^k}{k!}$$

Taylorův polynom exponenciální funkce stupně n v bodě nula.

$$\exp(x) \doteq T_n(x)$$

$\exp(x) = T_n(x) + R_n(x)$ ... $R_n(x) = \exp(x) - T_n$

Lagrangeův tvar zbytku pro $x = 1$



$$R_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(c)$$

$f(x) = \exp(x)$

$f^{(n+1)}(x) = \exp(x)$

$$R_n(x) = \frac{1}{(n+1)!} \exp(c)$$

$1 \le \exp(c) \le e$

$\frac{1}{(n+1)!} \to 0$ pro $n \to +\infty$

$x = 1$:

$$\underset{e}{\exp(1)} = T_n(1) + \underbrace{R_n(1)}_{\to 0 \text{ pro } n \to +\infty}$$

odtud:

$$e = \lim_{n \to +\infty} T_n(1) = \sum_{k=0}^{\infty} \frac{1}{k!}$$

Definice Taylorovy řady:

Nekonečnou řadu $\displaystyle\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

nazýváme Taylorovou řadou funkce f v bodě a.

$$\sum \frac{1}{k} = +\infty$$

$$\sum \frac{1}{k^{1+\varepsilon}}$$

$$\int_1^{+\infty} \frac{1}{x^{1+\varepsilon}} dx = \int_1^{+\infty} x^{-1-\varepsilon} dx =$$

$$= \left[-\frac{1}{\varepsilon} x^{-\varepsilon} \right]_1^{+\infty} = 0 + \frac{1}{\varepsilon} = \frac{1}{\varepsilon}$$

$$1 + \sum_{k=2}^{\infty} \frac{1}{k^{1+\varepsilon}} < 1 + \frac{1}{\varepsilon}$$