18) $y = x\sin x$ $\quad x\sin x = 0 \quad x_1 = 0$

$x_2 = \pi$

$$\int_0^\pi x\sin x\,dx = \left|\begin{matrix} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{matrix}\right| = [-x\cos x]_0^\pi + \int_0^\pi \cos x\,dx = [\sin x - x\cos x]_0^\pi =$$

$$= 0 + \pi - 0 = \pi$$

19) $y = \cos^5(x) \quad \cos^5 x = 0 \quad x_1 = -\frac{\pi}{2} \quad x_2 = \frac{\pi}{2}$

$$\int \cos^5(x)\,dx = \int \cos^4(x)\cdot\cos(x)\,dx = \int (\cos^2(x))^2\cos(x)\,dx = \int (1-\sin^2(x))^2\cos(x)\,dx = \left|\begin{matrix} t = \sin(x) \\ dt = \cos(x)dx \end{matrix}\right| =$$

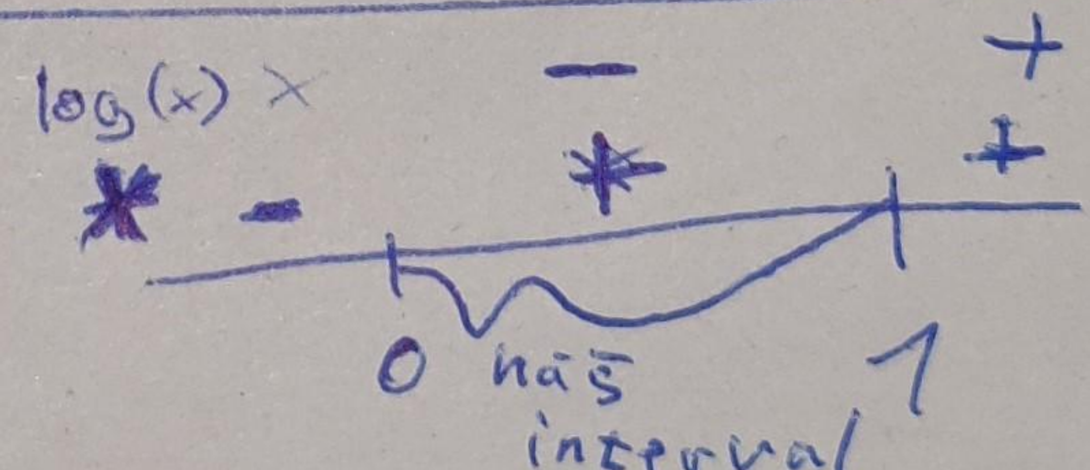
Tady jste se přepsal, na výsledku to nic nemění.

$$= \int (1-t^2)^2\,dt = \int (1-2t^2+t^4)\,dt = t \boxed{-\frac{2}{3}t^2} + \frac{1}{5}t^5 = \sin(x) - \frac{2}{3}\sin^2(x) + \frac{1}{5}\sin^5(x) + c$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^5(x)\,dx = \left[\sin(x) - \frac{2}{3}\sin^2(x) + \frac{1}{5}\sin^5(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right) = 2 + \frac{2}{5} - \frac{4}{3} = \frac{30+6-20}{15} = \frac{16}{15}$$

20) $y = x^2\log(x) \quad x^2\log(x) = 0 \quad x_1 = 0$

$x_2 = 1$



$$\int x^2\log(x)\,dx = \left|\begin{matrix} u = \log(x) & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{matrix}\right| = \frac{x^3\log(x)}{3} - \int \frac{x^3}{3x}\,dx = \frac{x^3\log(x)}{3} - \frac{1}{3}\int x^2\,dx = \frac{x^3\log(x)}{3} - \frac{x^3}{9} + c$$

$$\int_0^1 x^2\log(x)\,dx = \left[\boxed{\frac{x^3\log(x)}{3}} - \frac{x^3}{9}\right]_0^1 = 0 - \frac{1}{9} - 0 + 0 = -\frac{1}{9}$$

V nule je třeba spočítat limitu.

jelikož je plocha pod osou x, tak obsah vyšel záporný

$\Downarrow$

výsledek $\boxed{\frac{1}{9}}$