

$$5) f: \arctg\left(\frac{x}{x+1}\right)$$

$$x+1 \neq 0$$

$$x \neq -1$$

$$D(f) = \mathbb{R} \setminus \{-1\}$$

$$y = \arctg\left(\frac{x}{x+1}\right)$$

$$\frac{x}{x+1} = \operatorname{tg}(y)$$

$$x = x \operatorname{tg}(y) + \operatorname{tg}(y)$$

$$x - x \operatorname{tg}(y) = \operatorname{tg}(y)$$

$$x(1 - \operatorname{tg}(y)) = \operatorname{tg}(y)$$

$$f^{-1}: x = \frac{\operatorname{tg}(x)}{1 - \operatorname{tg}(x)}$$

~~$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \setminus \left\{\frac{\pi}{4}\right\}$$~~

$$1 - \operatorname{tg}(x) \neq 0$$

$$\operatorname{tg}(x) \neq 1$$

$$x \neq \arctg(1) = \frac{\pi}{4}$$

$$D(f^{-1}) = \left(-\frac{\pi}{2}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right) = H(f)$$

Výsledek je dobře až na to znaménko uprostřed. Postup by ale bylo dobré více komentovat (to platí i pro ostatní příklady).

~~$$6) f: y = \arcsin(x^2 + x)$$~~

~~$$x^2 + x \geq -1$$~~

~~$$x^2 + x \leq 1$$~~

~~$$x(x+1) \geq -1$$~~

~~$$x(x+1) \leq 1$$~~

~~$$\begin{cases} x \leq 1 \\ x \leq 0 \end{cases}$$~~

~~$$x \geq -1$$~~

~~$$x \geq 0$$~~

$$7) f: x \mapsto \exp\left(\frac{x+1}{x^2-1}\right)$$

$$x^2 - 1 \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

$$D(f) = \mathbb{R} \setminus \{\pm 1\}$$

~~$$\exp\left(\frac{x+1}{x^2-1}\right) = \exp\left(\frac{x+1}{(x+1)(x-1)}\right) = \exp\frac{1}{x-1}$$~~

Definiční obor lze rozšířit o -1.

$$\lim_{x \rightarrow 1^+} \exp\frac{1}{x-1} = \exp\left(\lim_{x \rightarrow 1^+} \frac{1}{x-1}\right) = \exp\frac{1}{+0} = \exp \infty = \infty$$

$$\lim_{x \rightarrow 1^-} \exp\frac{1}{x-1} = \exp\left(\lim_{x \rightarrow 1^-} \frac{1}{x-1}\right) = \exp\frac{1}{-0} = \exp(-\infty) = 0$$

Jednostranné limity se nerovnájí

⇓

nelze rozšířit o bod +1