BINOMICKÁ VĚTA

Nechť $n \in \mathbb{N} \cup \{0\}$, $a, b \in \mathbb{R}$.

Pak $$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Důkaz

matematickou indukcí

1) $n=0$ $\quad (a+b)^0 = 1$

$$\sum_{k=0}^{0} \binom{0}{k} a^k b^{n-k} = \binom{0}{0} a^0 b^0 = 1$$

2) indukční krok

předpokládáme: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

počtoc :

$$(a+b)^{n+1} = (a+b)^n \cdot (a+b) =$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} \cdot (a+b) =$$

$$= \sum_{\cancel{k}\, j=0}^{n} \binom{n}{\cancel{k}\, j} a^{\cancel{k}\, j+1} b^{n-\cancel{k}\, j} + \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k+1}$$

$$k = j+1$$

$$\sum_{k=1}^{n+1} \binom{n}{k-1} a^k b^{n-k+1} + \text{—''—}$$

$k=0$: $\quad k=n+1$

$$b^{n+1} + a^{n+1} + \sum_{\cancel{k=1}\; k=0}^{n \;\; n+1} a^k b^{n-k+1} \left(\underbrace{\binom{n}{k-1} + \binom{n}{k}}_{\binom{n+1}{k}} \right)$$