Jan 1983

jak je to v nule – 25.3. 1. prog

Ř $\int f(x)\,dx = F(x)$

víme: pro $x \neq 0$ je $\underline{F'(x) = f(x)}$

- $\lim\limits_{x \to 0} f(x) = f(0)$

- F je spojitá v nule

odtud plyne:

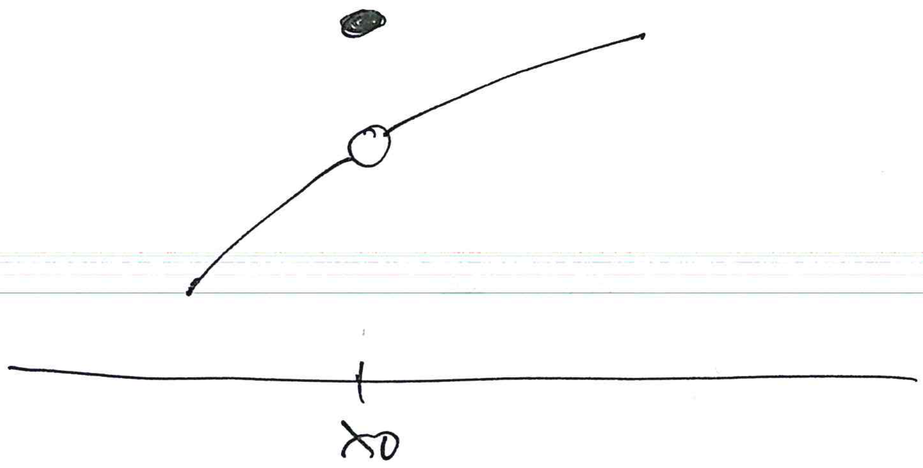
- ~~$\lim\limits_{x \to 0} F(x) = F(0)$~~

- $\lim\limits_{x \to 0} F'(x) = f(0)$

z věty ze 25.3., 1. prog plyne

F má v nule derivaci rovnu $f(0)$, tj. $\underline{F'(0) = f(0)}$

tedy $(\forall x \in \mathbb{R})(F'(x) = f(x))$



$\lim_{x \to x_0} f'(x)$ existuje, ale $f'(x_0)$ neexistuje

$$\int \frac{y}{y^2+2y+1}\,dy =$$

$$= \int \frac{y+\cancel{1}}{y^2+2y+1}\,dy - \int \frac{\cancel{1}}{y^2+2y+1}\,dy$$

$$= \underbrace{\frac{1}{2}\int \frac{2y+\cancel{2}}{y^2+2y+1}\,dy}_{} - \int \frac{\cancel{1}}{y^2+2y+1}\,dy$$

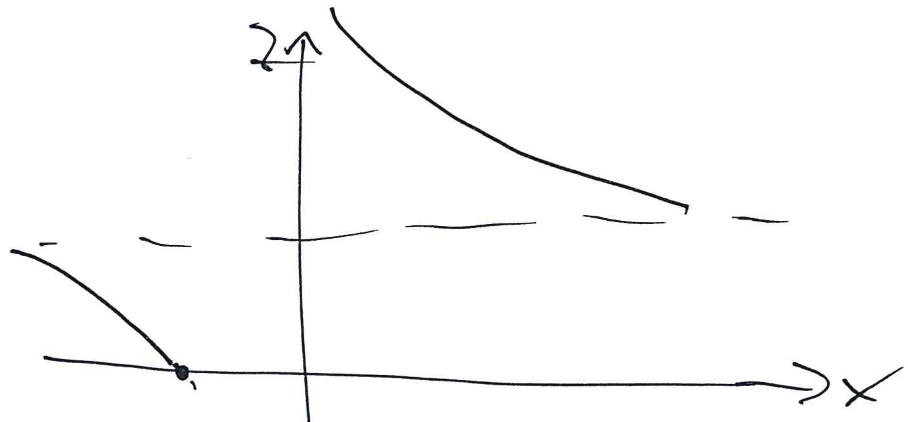
$$= \frac{1}{2}\log(y^2+2y+1)$$

$$\frac{y+\cancel{1}-1}{(y+1)^2} = \frac{A}{y+1} + \frac{B}{(y+1)^2}$$

$$\frac{1}{y+1} - \frac{1}{(y+1)^2}$$

$$\frac{y}{y^2+y+1} = \frac{A(2y+1)}{y^2+y+1} + \frac{B}{y^2+y+1}$$

$z = \frac{1}{x} + 1 \qquad y = \sqrt{z}$



$x \in (0, +\infty)$:

$z = (1, +\infty)$

$y \in (1, +\infty)$

$x \in (-\infty, -1)$:

$z \in (0, 1)$

$y \in (0, 1)$

$$\int \frac{y^2}{(y^2-1)^4}\,dy = \frac{1}{2}\int y \cdot \frac{2y}{(y^2-1)^4}\,dy$$

$$= \left(-\frac{1}{3}(y^2-1)^{-3}\right)'$$

$$= \frac{1}{2}\left(\frac{-y}{3(y^2-1)^{-3}}\right) + \frac{1}{3}\int \frac{1}{(y^2-1)^3}\,dy$$

$$\frac{1}{(y^2-1)^3} = \frac{A}{y^2-1} + \frac{By}{y^2-1} + \frac{C}{(y^2-1)^2} + \frac{Dy}{(y^2-1)^2}$$

$$+ \frac{E}{(y^2-1)^3} + \frac{Fy}{(y^2-1)^3}$$

$$E = 1$$

$$\frac{1}{y^2-1} = \frac{A}{y-1} + \frac{B}{y+1}$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$$

Jan 1983

Michal 1984

Václav 1981

Jan 1985 – Čebytěv

Michal 1982

$$\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$$

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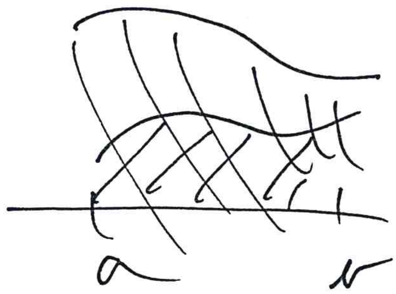
f je definovaná a Riemannovsky integrovatelná na $\langle a, b\rangle$:

$$(\forall x \in \langle a, b\rangle)\,\bigl(-|f(x)| \le f(x) \le |f(x)|\bigr)$$

Monotonie integrálu:

f, g R. I. na $\langle a, b\rangle$

$$(\forall x \in \langle a, b\rangle)\,\bigl(f(x) \le g(x)\bigr)$$



pak $\int_a^b f(x) \le \int_a^b g(x)$

odhad:

$$-\int_a^b |f(x)| \le \int_a^b f(x) \le \int_a^b |f(x)|$$

$$-B \le A \le B \iff A \le |B|$$

$\Leftarrow$ ~~[illegible]~~

$\Rightarrow$