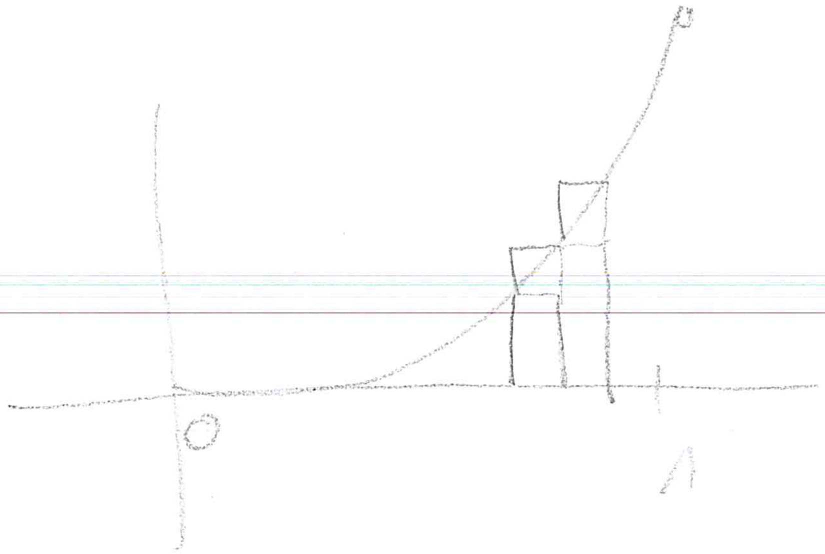


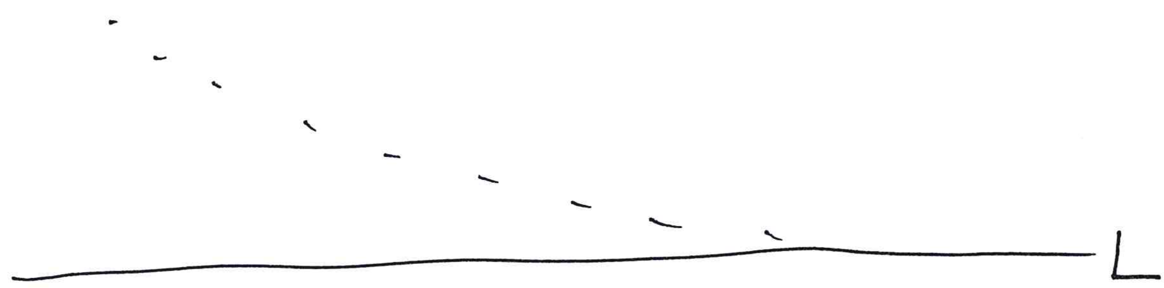
Tvrdím:

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^{p} \geq \overline{\int_0^1} x^p\,dx = \int_0^1 x^p\,dx$$

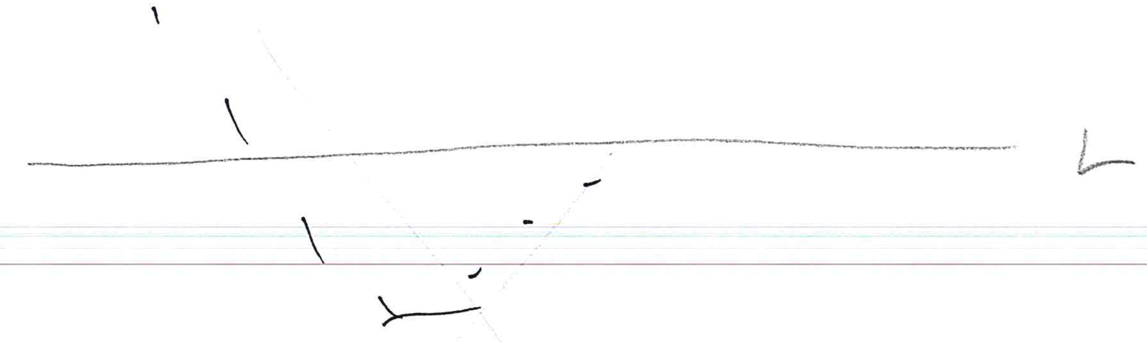
1) $$\inf_{n\in\mathbb{N}} \left\{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^{p}\right\}_{n\in\mathbb{N}} \geq \int_0^1 x^p\,dx$$

2) $$\lim_{n\to\infty} \frac{1}{n}\sum_{\substack{i=1\\0}}^{\substack{n-1\\n}}\left(\frac{i}{n}\right)^{p} \geq \inf\left\{\frac{1}{n}\sum_{\substack{i=1\\0}}^{\substack{n-1\\n}}\left(\frac{i}{n}\right)^{p}\right\}_{n\in\mathbb{N}}$$

$\leq \sup$



obecně: $$\underbrace{\lim_{n\to\infty} a_n}_{=L} \geq \inf\{a_n : n\in\mathbb{N}\}$$



$(\exists n \in \mathbb{N})(a_n < L)$ pak $L > \inf$

$(\forall n \in \mathbb{N})(a_n \geq L)$

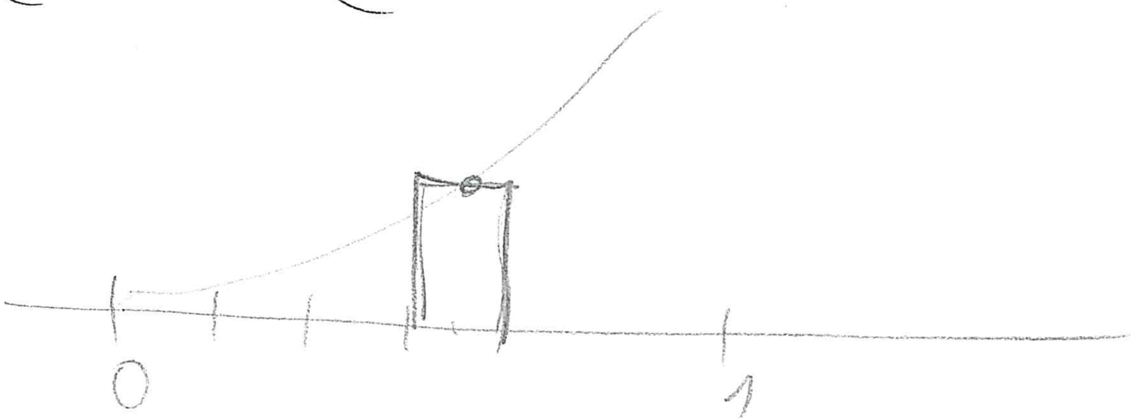
$(\forall \varepsilon > 0)(\exists n)(a_n < L + \varepsilon)$

odtud $L \geq \inf$

$$x^p dx \le \lim \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^p = \lim \frac{1}{n} \sum_{i=0}^{n-1} \left(\frac{i}{n}\right)^p \le \int_0^1 x^p dx$$

$$\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i-\frac{1}{2}}{n}\right)^p$$

$$\left(\frac{\frac{1}{2}}{n}\right)^p \qquad \left(\frac{\frac{3}{2}}{n}\right)^p$$



$$\frac{1^p+3^p+\dots+(2n-1)^p}{n^{p+1}}$$

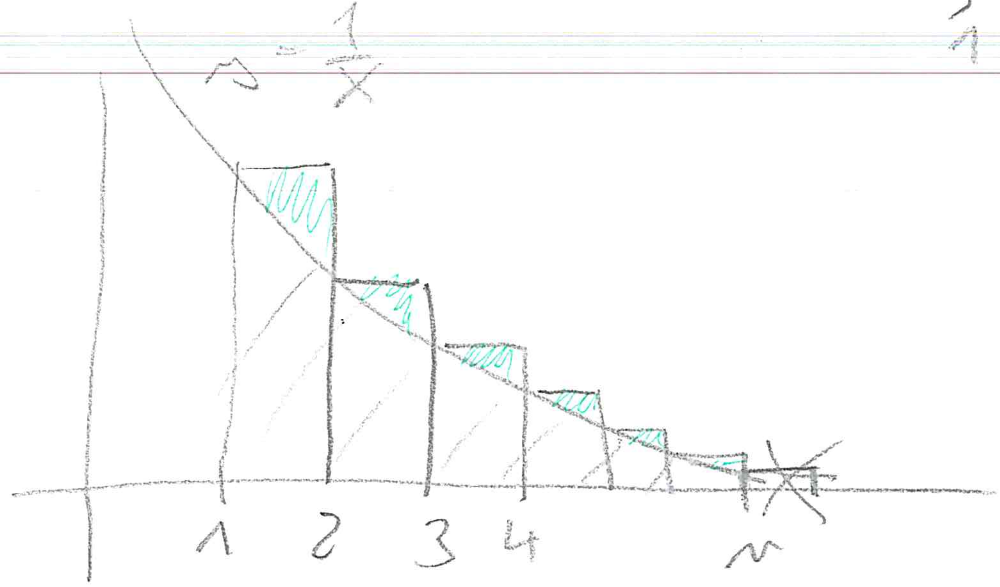
$$2^p\cdot\frac{1^p+2^p+\dots+n^p}{n^{p+1}}=\frac{2^p+4^p+6^p+\dots+(2n)^p}{n^{p+1}}$$

$$2^{p+1}\,\frac{1^p+2^p+3^p+\dots+(2n)^p}{(2n)^{p+1}}\to 2^{p+1}\cdot\frac{1}{p+1}$$

$$-2^p\cdot\frac{1}{p+1}$$

146

$$x_n = \sum_{k=1}^{n} \frac{1}{k} - \log n = \int_1^n \frac{1}{x}\,dx$$



$$y_n = \sum_{k=1}^{n-1} \frac{1}{k} - \log n$$

$$x_n = y_n + \frac{1}{n}$$

$$y_n = \sum_{k=1}^{n} \underbrace{\left(\frac{1}{k} - \int_k^{k+1} \frac{1}{x}\,dx \right)}_{\geq 0}$$

$$y_n = \sum_{k=1}^{n} \left(\frac{1}{k} - \log\left(1 + \frac{1}{k}\right) \right)$$

$$\log(1+x) = x + \ldots$$

1. 3 2.

Taylorův polynom stupně

Lagrangeův tvar zbytku