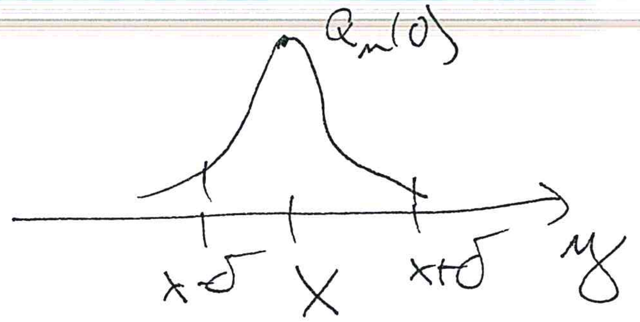
$$Q_n(\delta) = c_n(1-\delta^2)^n \le \sqrt{n}\,(1-\delta^2)^n$$

$$x-y \in (x-1,\, x-\delta) \Rightarrow Q_n(x-y) \le Q_n(\delta)$$



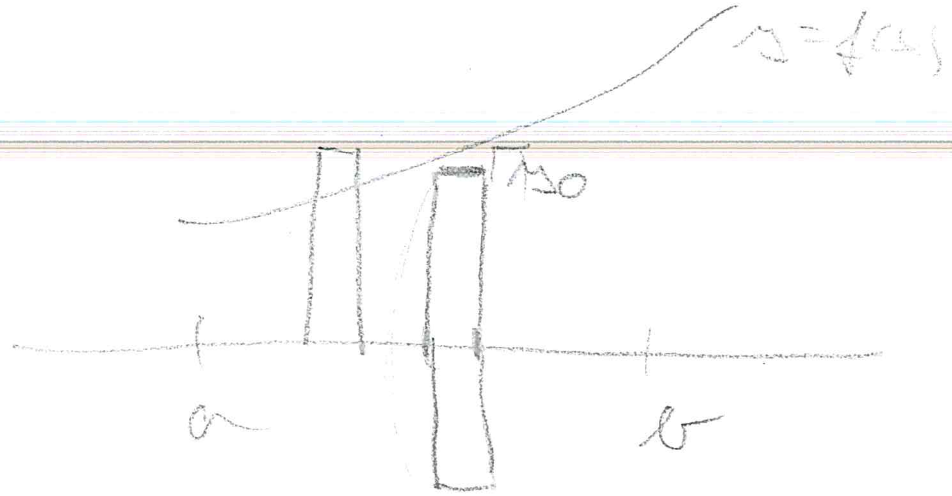
$$\int_{x-1}^{x-\delta} \underbrace{Q_n(x-y)}_{\le \sqrt{n}(1-\delta^2)^n} |f(x)-f(y)|\,dy \le$$

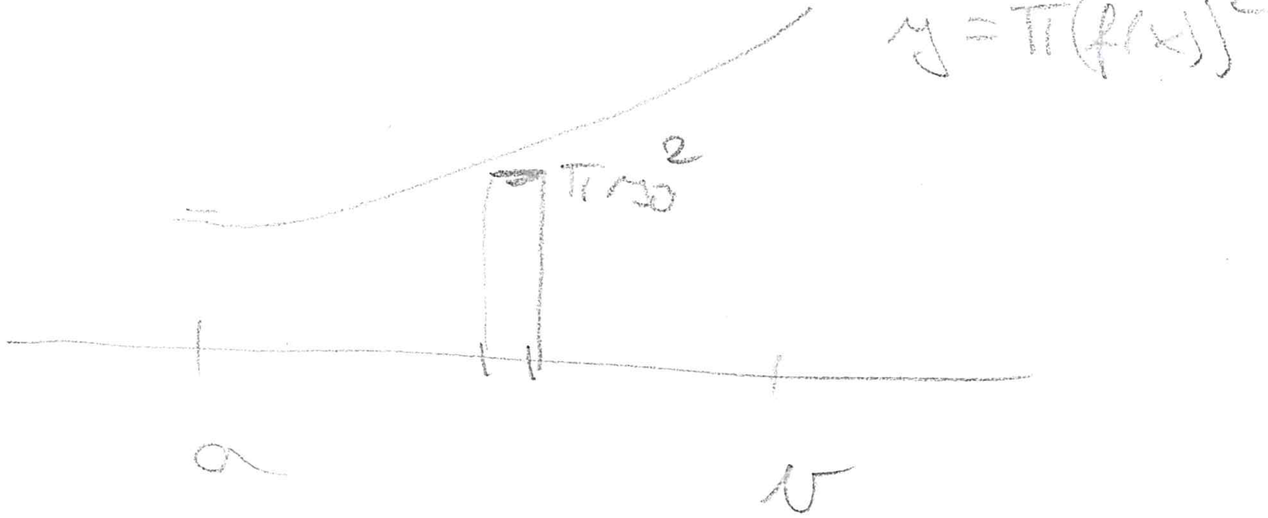
$$\le \sqrt{n}\,(1-\delta^2)^n \int_{x-1}^{x-\delta} \underbrace{|f(x)-f(y)|}_{\le M}\,dy \le$$

$$\le \sqrt{n}\,(1-\delta^2)^n M\,(1-\delta)$$

dnee: — ″ — $< \frac{\varepsilon}{3}$

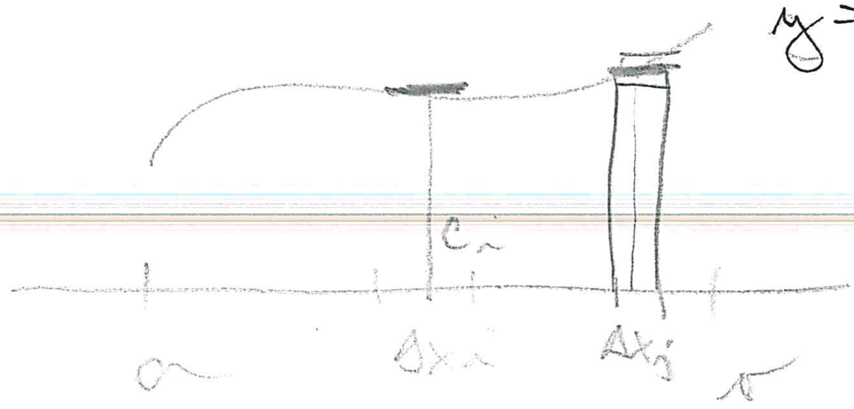
$$V = \int_a^b \pi (f(x))^2 \, dx$$





Těleso T, $\quad v_1 \subseteq T \subseteq v_2$

sjednocení válců



dělení D

$$DIS(D) \leq DL(D) \leq HIS(D)$$

$$\underline{\int_a^b} \sqrt{1+(f'(x))^2}\,dx = \sup_D \{DIS(D)\}$$

$$l = \sup \{DL(D)\}$$

odtud $\underline{\int_a^b} \sqrt{1+(f'(x))^2}\,dx \leq l$

chceme: $l \leq \overline{\int_a^b} \sqrt{1+(f'(x))^2}\,dx$

pak z $\underline{\int_a^b} \dots = \overline{\int_a^b} \dots$ plyne $l = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

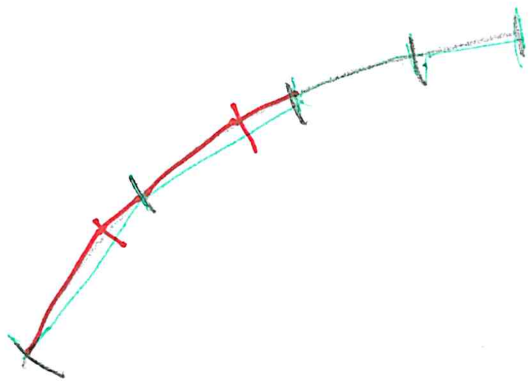
dvě dělení D_1, D_2, chceme:

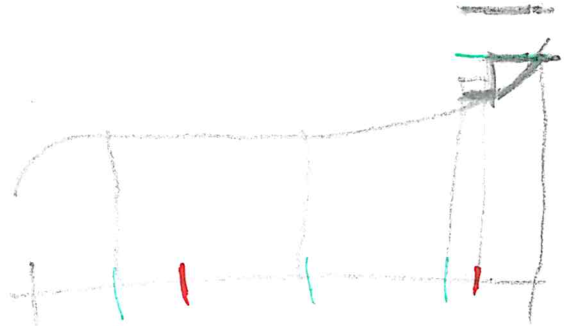
$$DL(D_1) \leq HIS(D_2)$$

D .. dělení obsahující uzly D_1 i D_2,

pak

$$DL(D_1) \leq DL(D) \leq HIS(D) \leq HIS(D_2)$$





Lema o oddělených množinách:

$$(\forall a \in A)(\forall b \in B)\,(a \leq b),$$

pak $\sup A \leq \inf B$

$$A = \{DL(D_1)\} \qquad B = \{HIS(D_2)\}$$

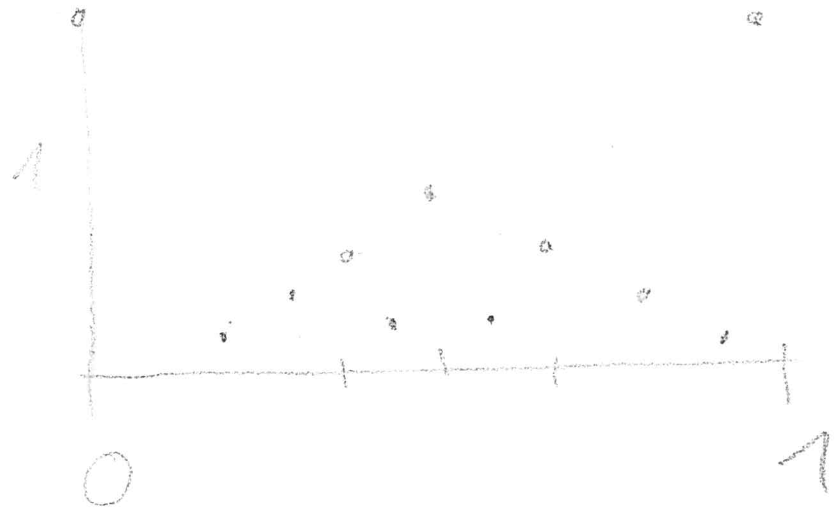
$$\sup A = l \qquad \inf B = \int_a^b \sqrt{1+(f'(t))^2}\,dt$$

tedy $$l \leq \int_a^b \sqrt{1+(f'(t))^2}\,dt$$

Riemannova funkce:

$$R(x) = \begin{cases} 0 & x \notin \mathbb{Q} \\ \frac{1}{q} & x = \frac{p}{q} \in \mathbb{Q} \end{cases}$$

ve zkráceném tvaru



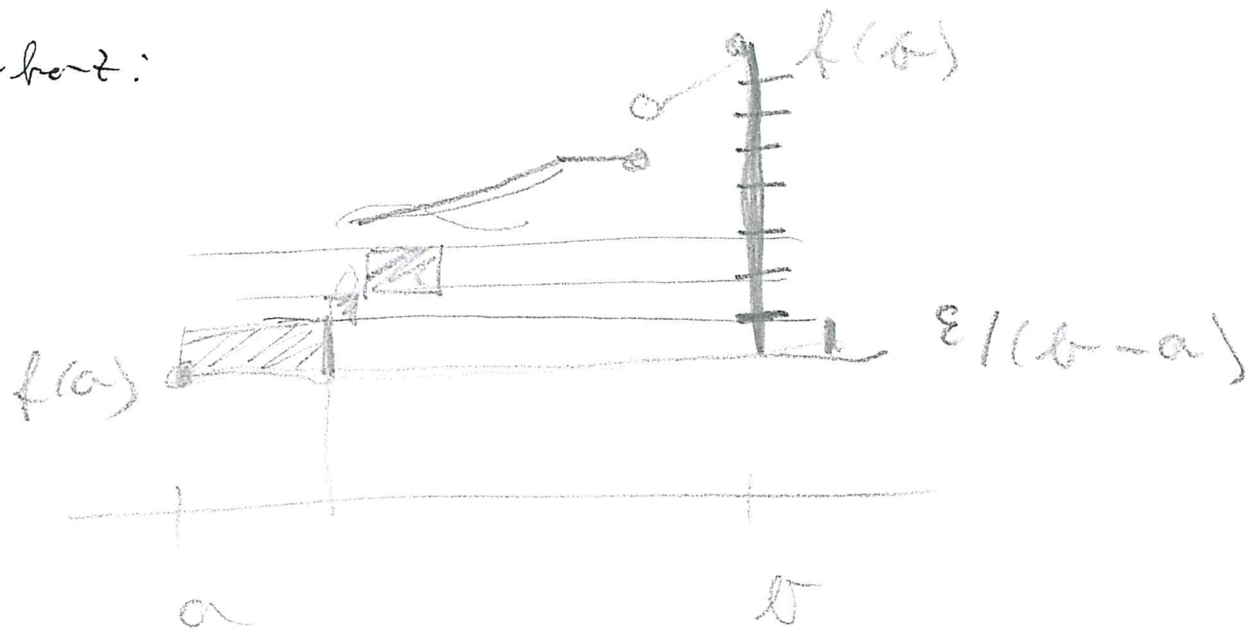
R je Riemannovsky integrovatelná na $[0,1]$

a $(R)\int_0^1 R(x)\,dx = 0$

Věta:

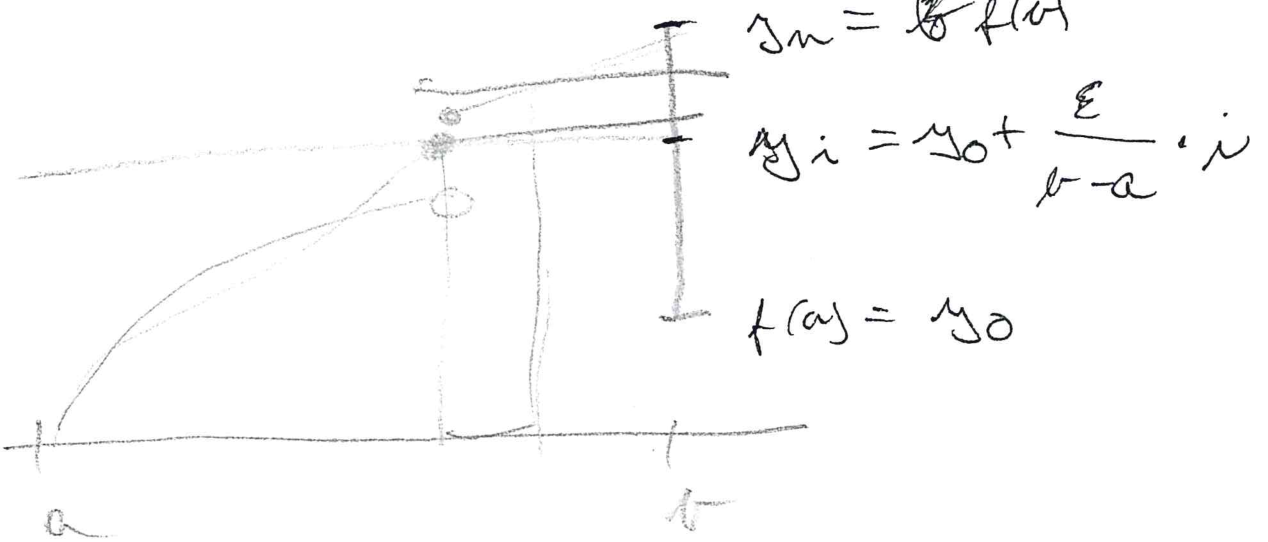
Nechť f je neklesající na $[a,b]$, pak je f Riemannovsky integrovatelná na $[a,b]$.

Důkaz:



$\forall \varepsilon > 0$

Konstruujeme DIS, HIS: HIS − DIS $< \varepsilon$



$$x_0 = a$$

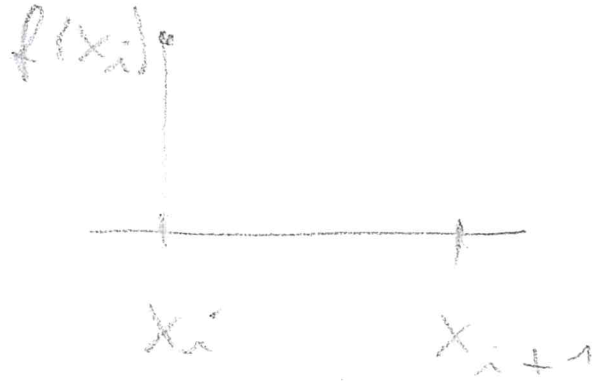
$$i \geq 1 \quad x_i = \sup \underbrace{\{x \in [a,b] : f(x) \leq y_i\}}_{M_i}$$

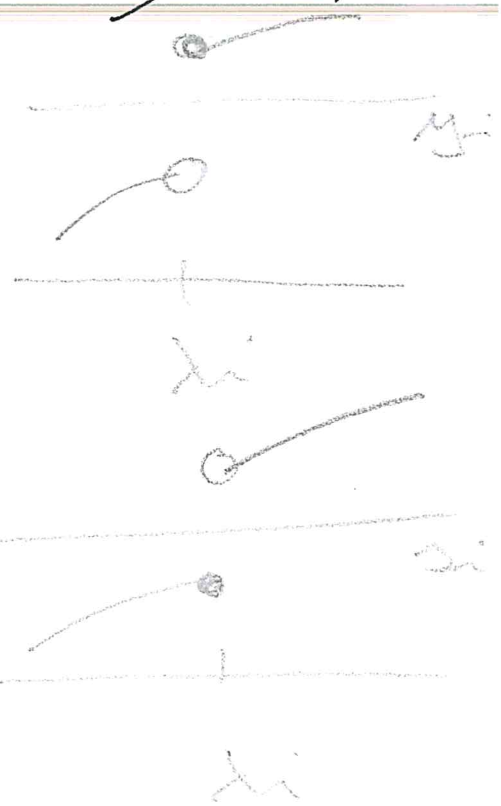
$a \in M$, tedy $M \neq \emptyset$

b je horní závora M, tedy M je omezená

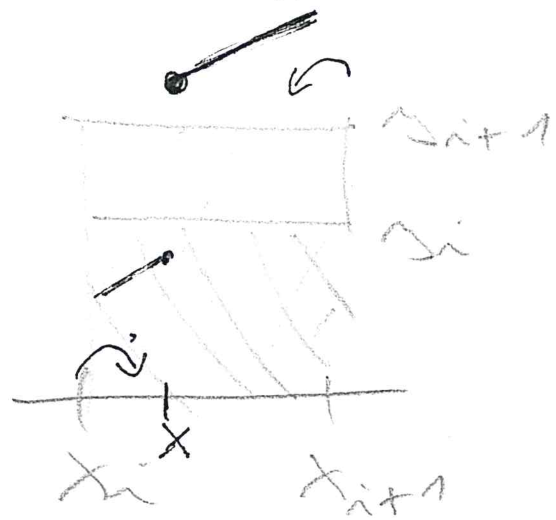
$$\boxed{x \in (x_i, x_{i+1}), \text{ pak } f(x) \in [y_i, y_{i+1}]}$$

$$x > x_i \qquad f(x) \geq f(x_i) \qquad \cancel{y_i \leq f(x_i)}$$





$$y_{i+1} - y_i \leq \frac{\varepsilon}{b-a}$$



$$f(x) > y_{i+1}$$

$$M_{i+1} \subseteq [a, x]$$

$$\sup M_{i+1} \leq x$$

$$HIS - DIS \leq \sum (x_{i+1} - x_i) \frac{\varepsilon}{b-a} =$$

$$= \frac{\varepsilon}{b-a} \sum (x_{i+1} - x_i) = \varepsilon$$