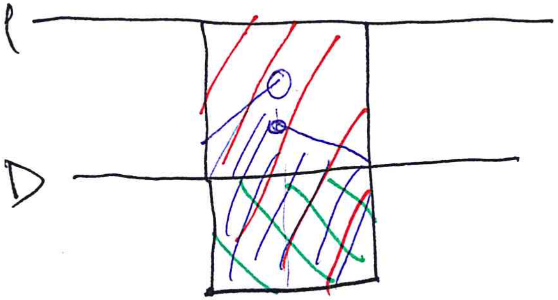
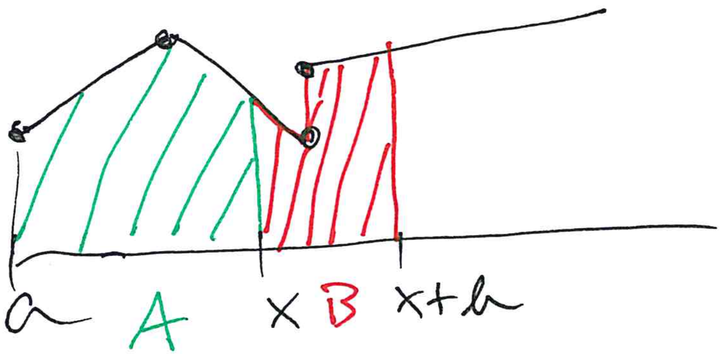
Co používáme:

① $A \subseteq B \Rightarrow$ obsah$(A) \leq$ obsah(B)



$$\sigma \leq \sigma \leq \sigma$$

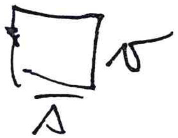
② $A \cap B = \emptyset \Rightarrow$ obsah$(A \cup B) =$ obsah$(A) +$ obsah(B)



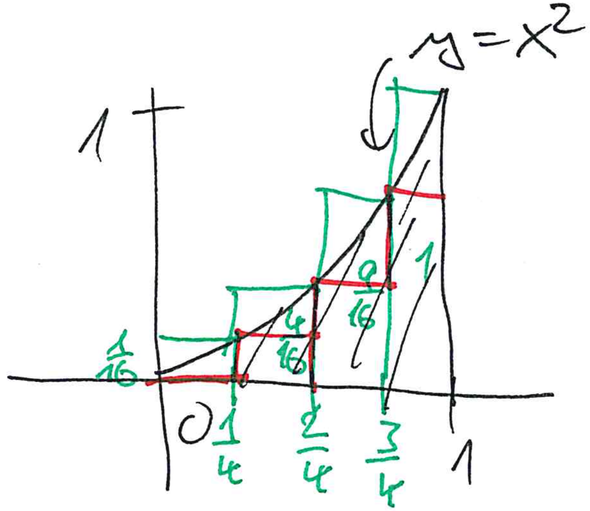
obsah(A) obsah$(A \cup B)$

$\sigma(x)$ $\sigma(x+h)$

obsah$(B) = \sigma(x+h) - \sigma(x)$

③ obsah obdélníku 

$\sigma = \bar{s} \cdot v$



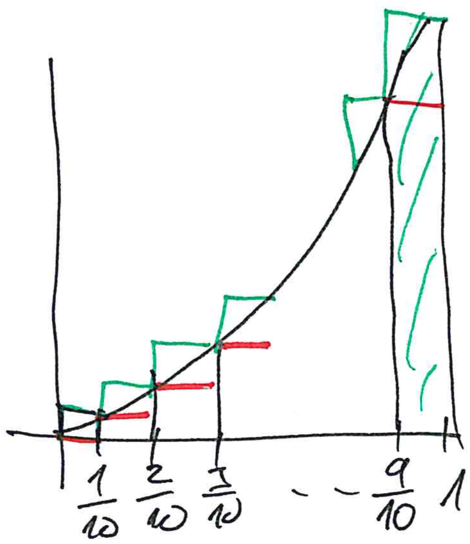
$$\sigma(\Delta) = ?$$

$$\frac{1}{4}\left(\frac{1}{16}+\frac{4}{16}+\frac{9}{16}+\frac{16}{16}\right) \le \sigma(\Delta) \le \frac{15}{32}$$

$$\cancel{\frac{15}{32}}$$

$$\frac{1}{4}\left(0+\frac{1}{16}+\frac{4}{16}+\frac{9}{16}\right)$$

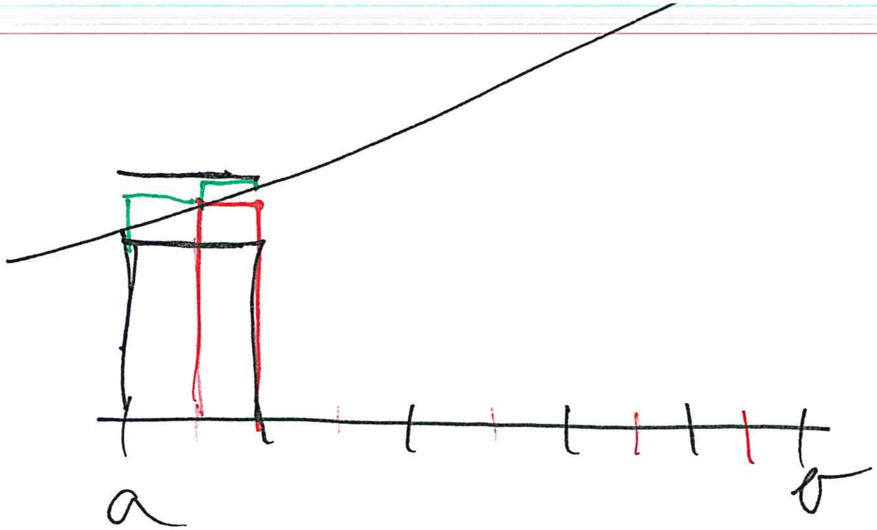
$$\frac{7}{32}$$



$$\frac{1}{10}\left(0+\frac{1}{100}+\frac{4}{100}+\frac{9}{100}+\dots\right) \le \sigma(\Delta) \le \frac{385}{1000}$$

$$\frac{1+4+9+16+25+36+49+64+81 \quad +100}{1000}$$

$$\frac{285}{1000} \qquad 0.285 \qquad 0.385$$



Věta:

Je-li f spojitá na $[a,b]$, pak má na $[a,b]$ Riemannův integrál.

Pojem: Integrál s proměnnou horní mezí

$$R(x) = (R)\int_a^x f(t)\,dt$$

Věta: Má-li f Riemannův integrál na $[a,b]$ a f je spojitá v bodě $x \in (a,b)$, pak $R'(x) = f(x)$

Důsledky:

① Je-li f spojitá na $[a,b]$, pak je R primitivní funkcí na (a,b).

② Je-li f spojitá na (a,b), pak existuje k f primitivní funkce na (a,b).

③ Je-li f spojitá na $[a,b]$, pak existují $(R)\int_a^b f(x)\,dx$, $(N)\int_a^b f(x)\,dx$ a rovnají se.

Proto píšeme $\int_a^b f(x)\,dx$

$$\sigma(\Delta) = \int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$