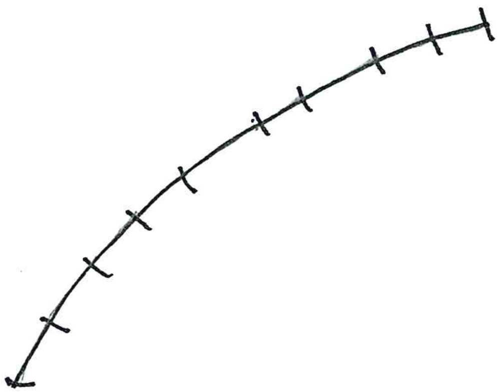
DÉLKA KŘIVKY

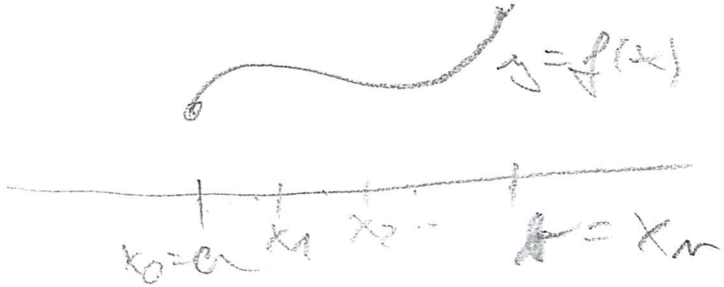


Do křivky vepíšeme lomenou čáru.
Délka této lomené čáry je spodní (skládá se z úseček)
odhad délky křivky.

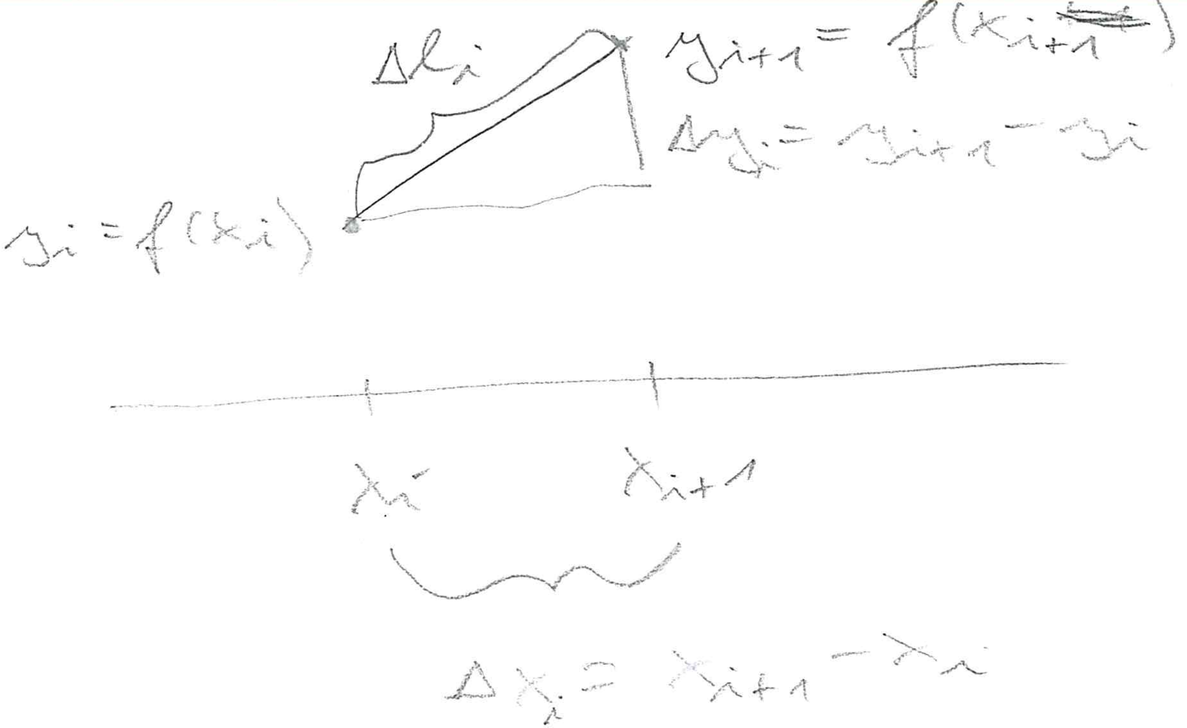
Definice:

Nechť f je definovaná na intervalu $[a, b]$.

Délkou křivky $K = \{[x, y] : x \in [a, b],\ y = f(x)\}$



nazýváme supremum délek lomených čar vepsaných do této křivky. $([x_0, f(x_0)] - [x_1, f(x_1)] - \dots - [x_n, f(x_n)])$



$$\Delta l_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} =$$

$$= \Delta x_i \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}$$

$$\approx f'(x)$$

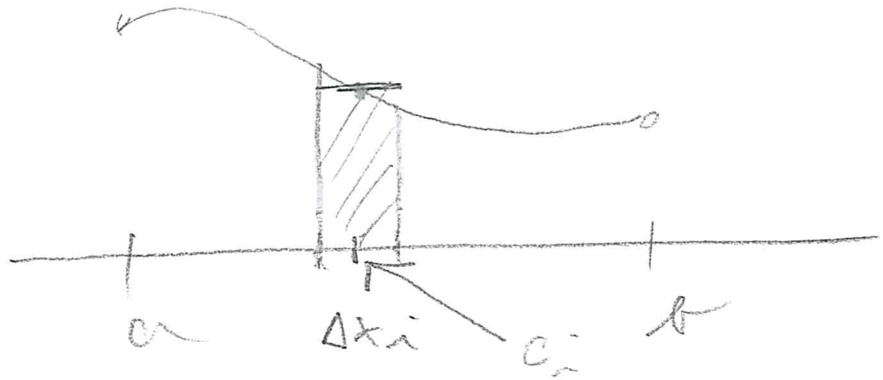
$$= f'(c_i)$$

z Lagrangeovy věty – kde

$$\exists\, c_i \in (x_i, x_{i+1})$$

$$l = \sum_{i=0}^{n-1} \Delta l_i = \sum_{i=0}^{n-1} \Delta x_i \sqrt{1 + \left(f'(c_i)\right)^2}$$

graf funkce $g(x) = \sqrt{1+(f'(x))^2}$



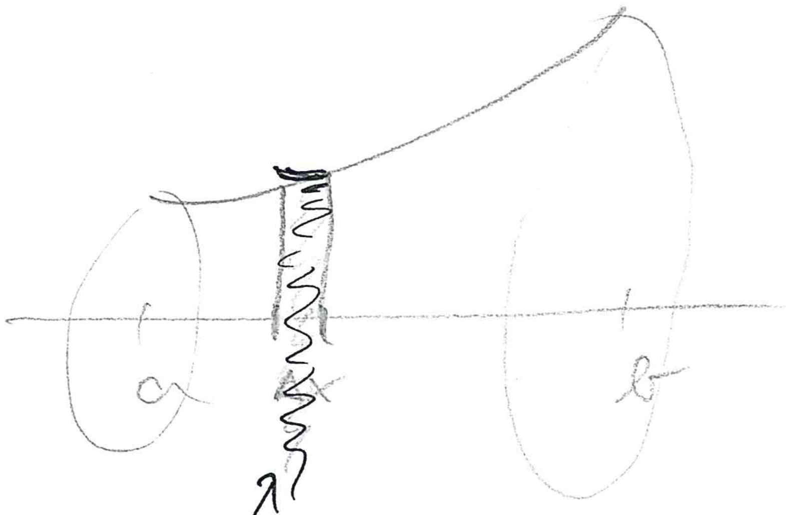
Intuice:

$$L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

Jinak (z dob Newtona a Leibnize)

$$\frac{dL}{dx} = \sqrt{1+(f'(x))^2} \quad \text{odtud} \quad L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

OBJEM ROTAČNĚ SYMETRICKÉHO TĚLESA



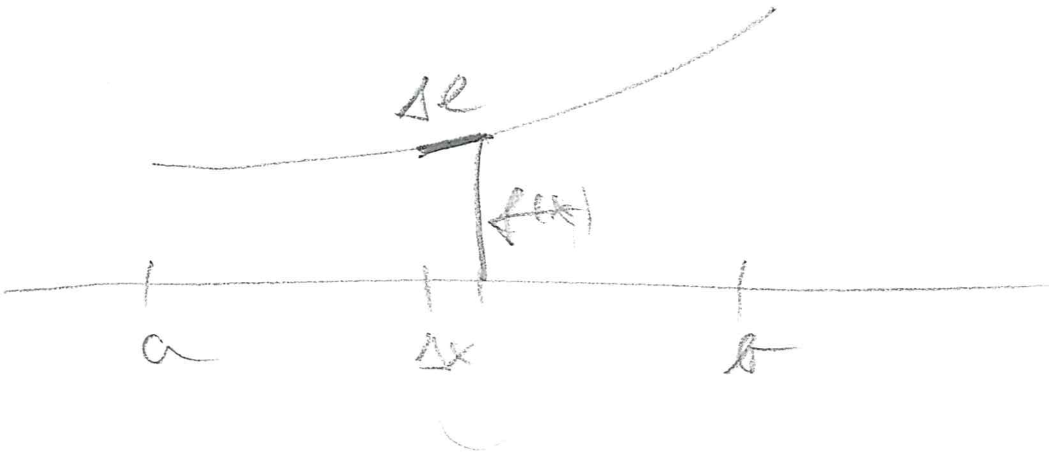
Podle Newtona a Leibnize:

$$\Delta V \doteq \pi \left(f(x)\right)^2 \Delta x$$

$$dV = \pi \left(f(x)\right)^2 dx$$

$$V = \int_a^b \pi \left(f(x)\right)^2 dx$$

POVRCH PLÁŠTĚ ROTAČNĚ SYMETRICKÉHO TĚLESA



$$\Delta S \doteq \Delta l \cdot 2\pi f(x) = \Delta x \sqrt{1+(f'(c))^2} \cdot 2\pi f(x)$$

$$dS \doteq 2\pi f(x) \sqrt{1+(f'(x))^2}\, dx$$

$$S = \int_a^b 2\pi f(x) \sqrt{1+(f'(x))^2}\, dx$$

$$\int \sin^2 x \, dx \; :$$

① použijeme vzorec

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\int \sin^2(x)\,dx = \int \frac{1-\cos(2x)}{2}\,dx = \int \frac{1}{2} - \frac{\cos(2x)}{2}\,dx =$$

$$= \frac{1}{2}x - \frac{\sin(2x)}{4}$$

(2) integrace per partes

$$I = \int \sin^2(x)\,dx = \int \sin(x)\cdot\sin(x)\,dx = \left[\begin{array}{ll} f = \sin(x) & g' = \sin(x) \\ f' = \cos(x) & g = -\cos(x) \end{array}\right] =$$

$$= -\sin(x)\cdot\cos(x) + \int \cos^2(x)\,dx =$$

$$= -\sin(x)\cos(x) + \int 1 - \sin^2(x)\,dx =$$

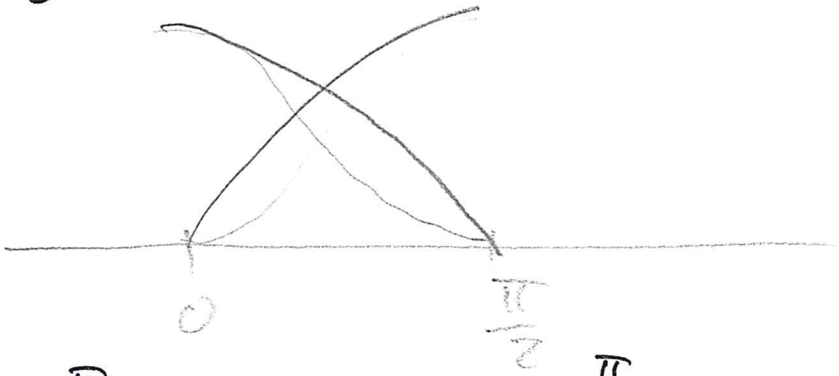
$$= -\sin(x)\cos(x) + x - \int \sin^2(x)\,dx = -\sin(x)\cos(x) + x - I$$

z rovnice $$I = -\sin(x)\cos(x) + x - I$$

vyjádříme $$I = -\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}$$

3. $$\int_0^{\frac{\pi}{2}} \sin^2(x)\,dx$$

podstatné – meze jsou $k\cdot\frac{\pi}{2}$, $k\in\mathbb{Z}$



$$I = \int_0^{\frac{\pi}{2}} \sin^2(x)\,dx = \int_0^{\frac{\pi}{2}} \cos^2(x)\,dx$$

$$I + I = \int_0^{\frac{\pi}{2}} \underbrace{\sin^2(x) + \cos^2(x)}_{=1}\,dx = \ldots\ldots = \frac{\pi}{2}$$

odhad: $\quad I = \frac{\pi}{4}$