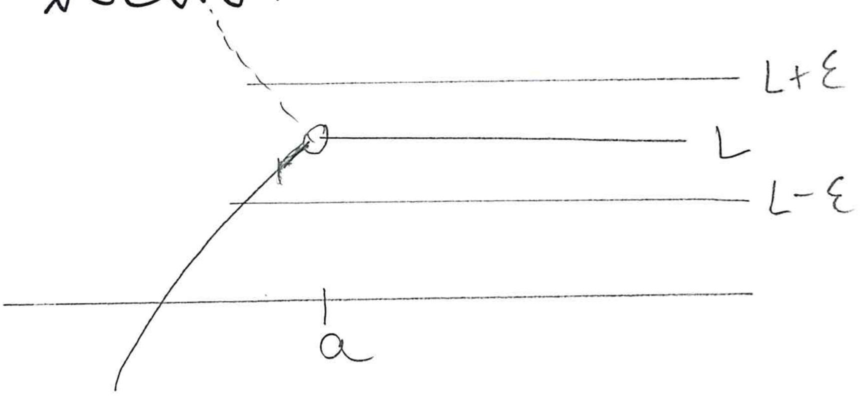
LIMITA MONOTONNÍ FUNKCE

Věta:

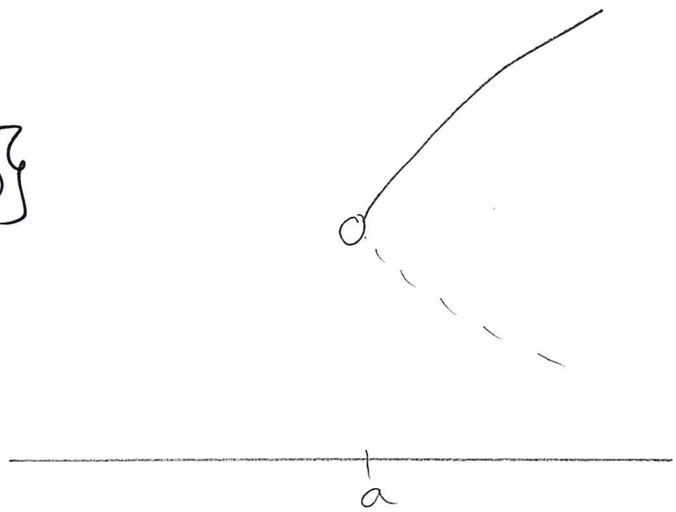
Je-li funkce g monotonní v pravém | levém okolí bodu a, pak má g v bodě a zprava / zleva limitu.

(Limita je v tomto případě rovna supremu nebo infimu funkčních hodnot v daném okolí.)



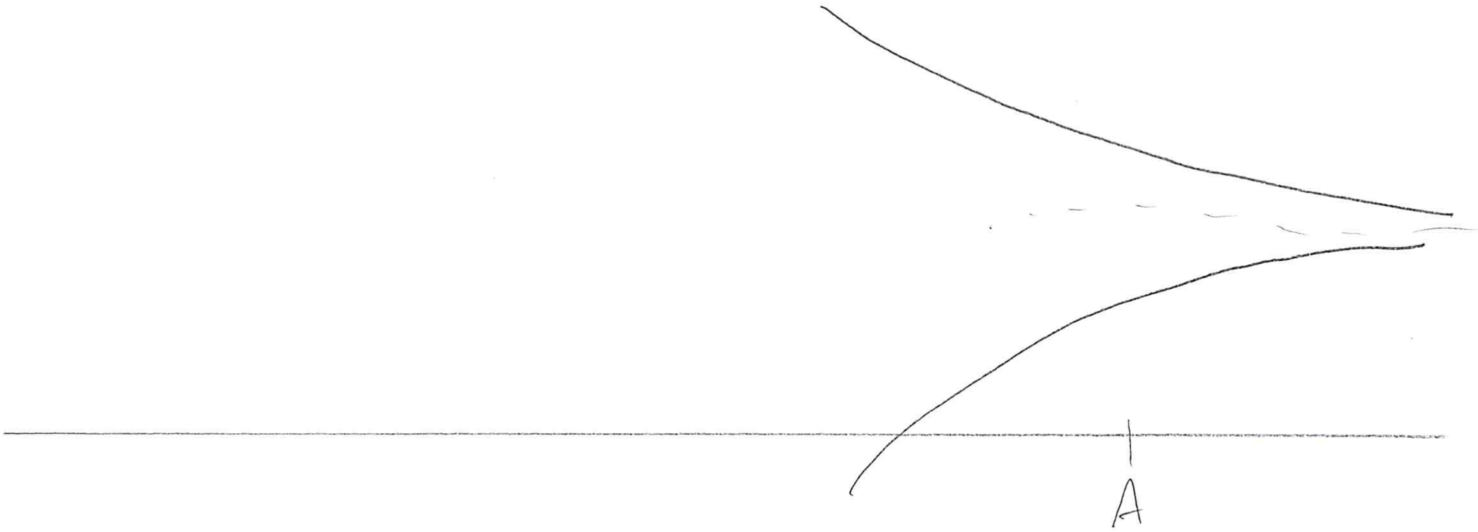
$$\lim_{x\to a^-} g(x) = \sup\{g(x) : x\in(a-\delta, a)\}$$

inf



$$\lim_{x\to a^+} g(x) = \inf\{g(x) : x\in(a, a+\delta)\}$$

sup

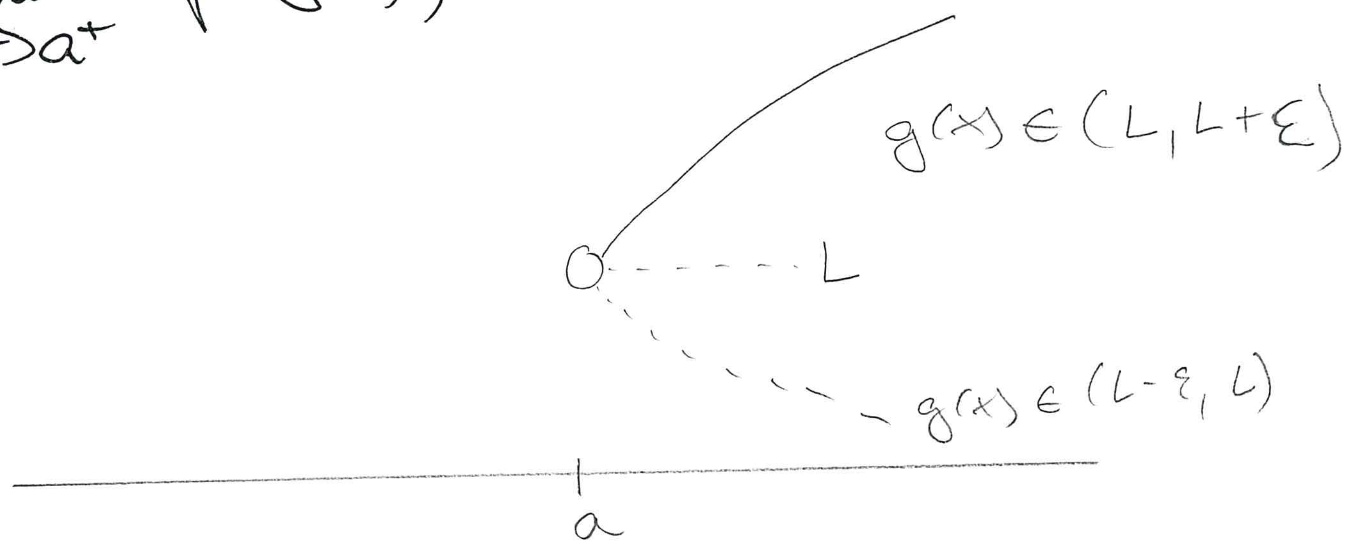


$$\lim_{x\to+\infty} g(x) = \inf\{g(x) : x\in(A,+\infty)\}$$

sup

LIMITA SLOŽENÉ FUNKCE S MONOTONNÍ VNITŘNÍ FUNKCÍ

$$\lim_{x \to a^+} f(g(x))$$



$$\lim_{x \to a^+} f(g(x)) = \lim_{y \to L^+} f(y)$$

$$y \to L^-$$

Spojité rozšíření funkce

$$f(x) = \exp\left(-\frac{1}{x^2}\right)$$

$$f(0) = 0$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h - 0} = \lim_{h \to 0} \frac{\exp\left(-\frac{1}{h^2}\right)}{h} =$$

$$= \lim_{h \to 0} \frac{1}{h} \exp\left(-\frac{1}{h^2}\right) = \qquad y = \frac{1}{h}$$

$$\lim_{h \to 0^\pm} \frac{1}{h} \exp\left(-\frac{1}{h^2}\right) = \lim_{y \to \pm\infty} y \exp\left(-y^2\right) =$$

$$= \lim_{y \to \pm\infty} \frac{y}{\exp(y^2)} \qquad \text{"}\frac{\infty}{\infty}\text{"}$$

$$L'H: \lim_{y \to \pm\infty} \frac{1}{2y \exp(y^2)} = 0$$

Závěr: $f'(0) = 0$

$$f_1(x) = \sin\left(\frac{1}{x}\right)$$

$$f_2(x) = x \sin\left(\frac{1}{x}\right)$$

$$-|x| \leq x \sin\left(\frac{1}{x}\right) \leq |x|$$

0 pro $x \to 0$

odkud

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$$

$$f_2'(0) = \lim_{h \to 0} \frac{h \sin \frac{1}{h}}{h} = \lim_{h \to 0} \sin \frac{1}{h}$$

f_2' v nule neexistuje

$$f_3(x) = x^2 \sin \frac{1}{x}$$

$$f_3(0) = 0$$

$$f_3'(0) = \lim_{h \to 0} \frac{h^2 \sin \frac{1}{h}}{h} =$$

$$= \lim_{h \to 0} h \sin \frac{1}{h} = 0$$

$$f_3'(0) = 0$$

$$x \neq 0: \quad f_3'(x) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}$$

Závěr:

f_3 je spojitá na $\mathbb{R}$

f_3 má derivaci na $\mathbb{R}$

f_3' není spojitá na $\mathbb{R}$