

$$\Delta_n = \sum_{k=1}^n a_k$$

$$\sum_{k=m}^N a_k b_k = a_m b_m + \dots + a_N b_N =$$

$$= (\Delta_m - \Delta_{m-1}) b_m + \dots + (\Delta_N - \Delta_{N-1}) b_N =$$

$$= -\Delta_{m-1} b_m + \Delta_m (b_m - b_{m+1}) + \dots + \Delta_N b_N$$

$-k \leq \Delta_m \leq k \quad | \cdot (b_m - b_{m+1}) > 0$

$$-2k b_m \leq \sum_{k=m}^N a_k b_k \leq 2k b_m$$

$$(\forall n \in \mathbb{N}) (|\Delta_n| < k)$$

oddhodly plyne, ze $\left\{ \sum_{k=1}^n a_k b_k \right\}_{n=1}^{\infty}$ je

Cauchyovská posloupnost, a tedy je konvergentní

a z $\boxed{b_n \downarrow 0}$