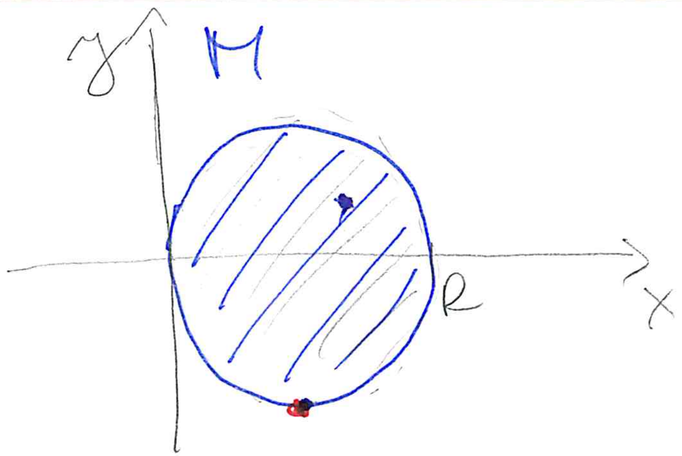
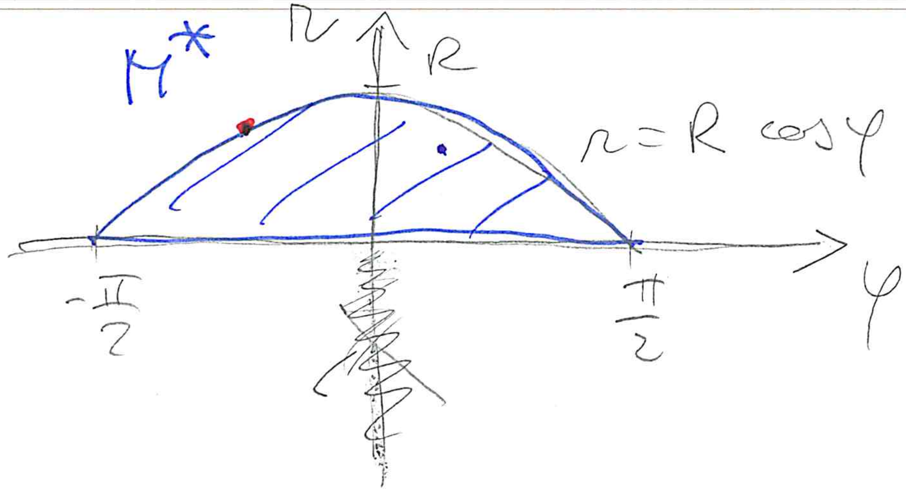




$$M^* = \{[\varphi, r] \in \cancel{[0, 2\pi)} \times [0, +\infty) : [r\cos\varphi, r\sin\varphi] \in M\}$$

$$(-\pi, \pi]$$

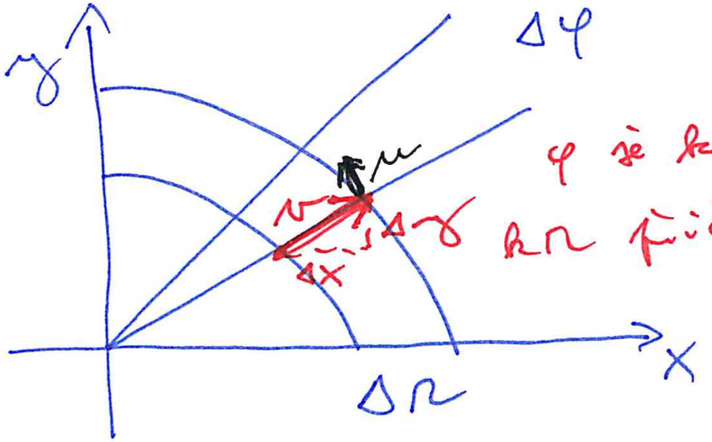
$$\iint_M f(x,y)\,dx\,dy = \iint_{M^*} f(r\cos\varphi, r\sin\varphi) \cdot r \cdot dr\,d\varphi$$

1) získáme elementární geometrií

nebo

2) Jacobián

pravidlo substituce ve dvojném integrálu



φ je konstatní
k r přičtu Δr

$$v = (\Delta x, \Delta y) = \Delta r \left(\frac{\Delta x}{\Delta r}, \frac{\Delta y}{\Delta r} \right)$$

$$\Delta r \to 0 \qquad dr \left(\frac{\partial x}{\partial r}, \frac{\partial y}{\partial r} \right)$$

$$u = (\Delta x, \Delta y) = \Delta \varphi \left(\frac{\Delta x}{\Delta \varphi}, \frac{\Delta y}{\Delta \varphi} \right)$$

$$\Delta \varphi \to 0 : \qquad d\varphi \left(\frac{\partial x}{\partial \varphi}, \frac{\partial y}{\partial \varphi} \right)$$

Jacobiova matice zobrazení $(r,\varphi) \mapsto (x,y) = (r\cos\varphi, r\sin\varphi)$

transformace z ~~ke~~ polárních do kartézských souřadnic

$$J = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} \end{pmatrix}$$

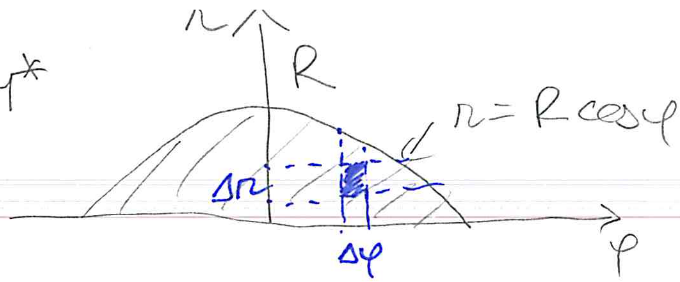
Jacobián – determinant Jacobiovy matice

$$\det J = \det \begin{pmatrix} \cos\varphi & \sin\varphi \\ -r\sin\varphi & r\cos\varphi \end{pmatrix} = r$$

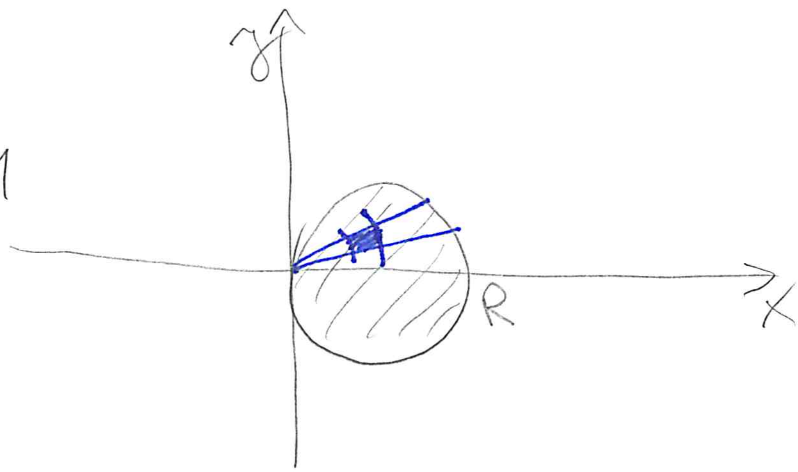
opakování z algebry

$$\det \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$$

M^*



M



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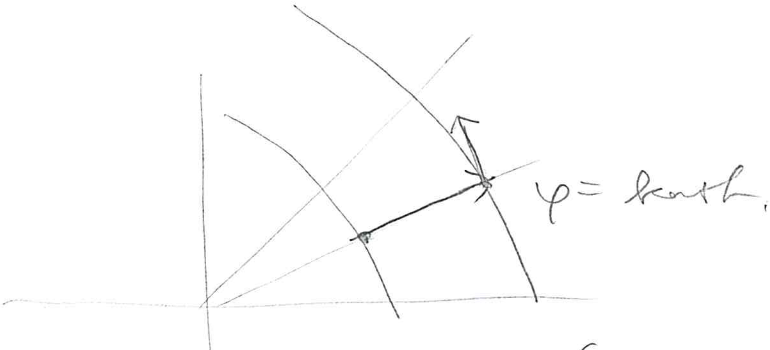


$v = (\Delta x, \Delta y) = \Delta r \left(\frac{\Delta x}{\Delta r}, \right.$

$dr \left(\frac{\partial x}{\partial r}, \frac{\partial}{\partial} \right.$

$w = \Delta \varphi \left(\frac{\Delta x}{\Delta \varphi}, \frac{\Delta y}{\Delta \varphi} \right)$

$d\varphi \left(\frac{\partial x}{\partial \varphi}, \frac{\partial y}{\partial \varphi} \right)$

Pravidlo o substituci:

$$\iint_M f(x,y)\,dx\,dy = \iint_{M^*} f(x(u,v), y(u,v))\,|\det J|\,du\,dv$$

$$M^* = \{[u,v] : [x(u,v), y(u,v)] \in M\}$$

$$J = \begin{pmatrix} \frac{\partial x}{\partial u}, & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u}, & \frac{\partial y}{\partial v} \end{pmatrix}$$

M:

~~M^*:~~ $u = xy$

$v = \frac{y}{x}$

$$M^*: \begin{cases} v \in [\frac{1}{2}, 2] \\ u \in [0, 1] \end{cases}$$

$S = \iint 1 \cdot |\det J|\,du\,dv$ meze

$$\det\begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix} = \det\begin{pmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x}\end{pmatrix} = y\cdot\frac{1}{x} + \frac{y}{x^2}\cdot x = 2\frac{y}{x}$$

$$= 2v$$

det inverzní matice: $\frac{1}{2v}$

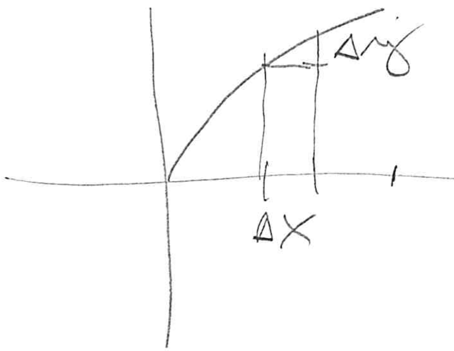
(opakování algebry)

Transformace: Inverzní transformace

$(x,y)\mapsto(u,v)$ $(u,v)\mapsto(x,y)$

pravidlo pro derivaci ~~složené fce~~ inverzní fce

$$\begin{pmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{pmatrix} = \begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix}^{-1}$$



$y = f(x)$

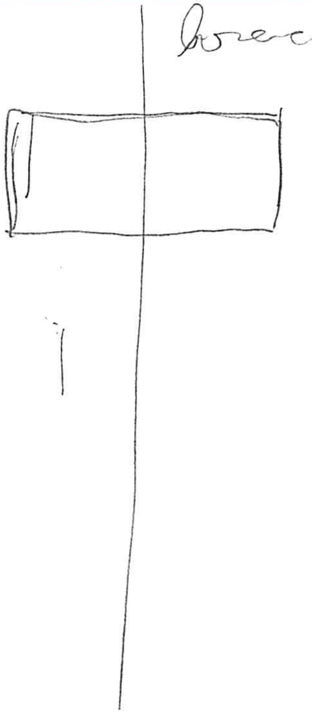
$y' \doteq \frac{\Delta y}{\Delta x}$

$x = f^{-1}(y)$

$$x' = \frac{\Delta x}{\Delta y} = \frac{1}{\frac{\Delta y}{\Delta x}} = \frac{1}{f'(x)}$$

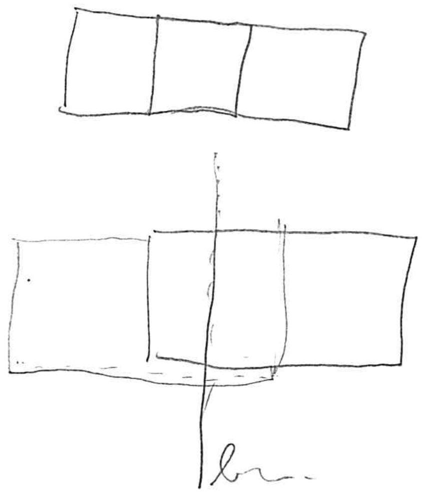
Stůl

hrana stolu



1 karta

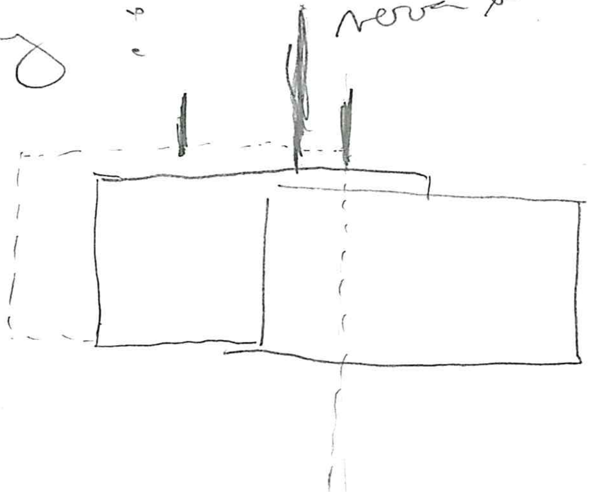
2 karty



hr...

3 karty:

nová hrana stolu?



1 karta

2. karty

Těžiště