

$$I = \int_{-\infty}^{+\infty} e^{-x^2} dx = 2$$

$$I^2 = I \cdot I = \int_{-\infty}^{+\infty} e^{-x^2} dx \cdot \int_{-\infty}^{+\infty} e^{-y^2} dy =$$

$$= \iint_{\mathbb{R}^2} e^{-x^2-y^2} dx dy$$

↖ polární souřadnice

$r \in$

$\varphi \in$

$$\int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} e^{-r^2} r dr \right) d\varphi$$

$$\int_{-\infty}^{+\infty} \left[-\frac{1}{2} e^{-r^2} \right]_{-\infty}^{+\infty} d\varphi$$

$$\int_{-\infty}^{+\infty} e^{-x^2} dx$$

$$\int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$$\sqrt{2\pi} \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt$$

$$\frac{t}{\sqrt{2}} = x$$

$$e^{-x^2}$$