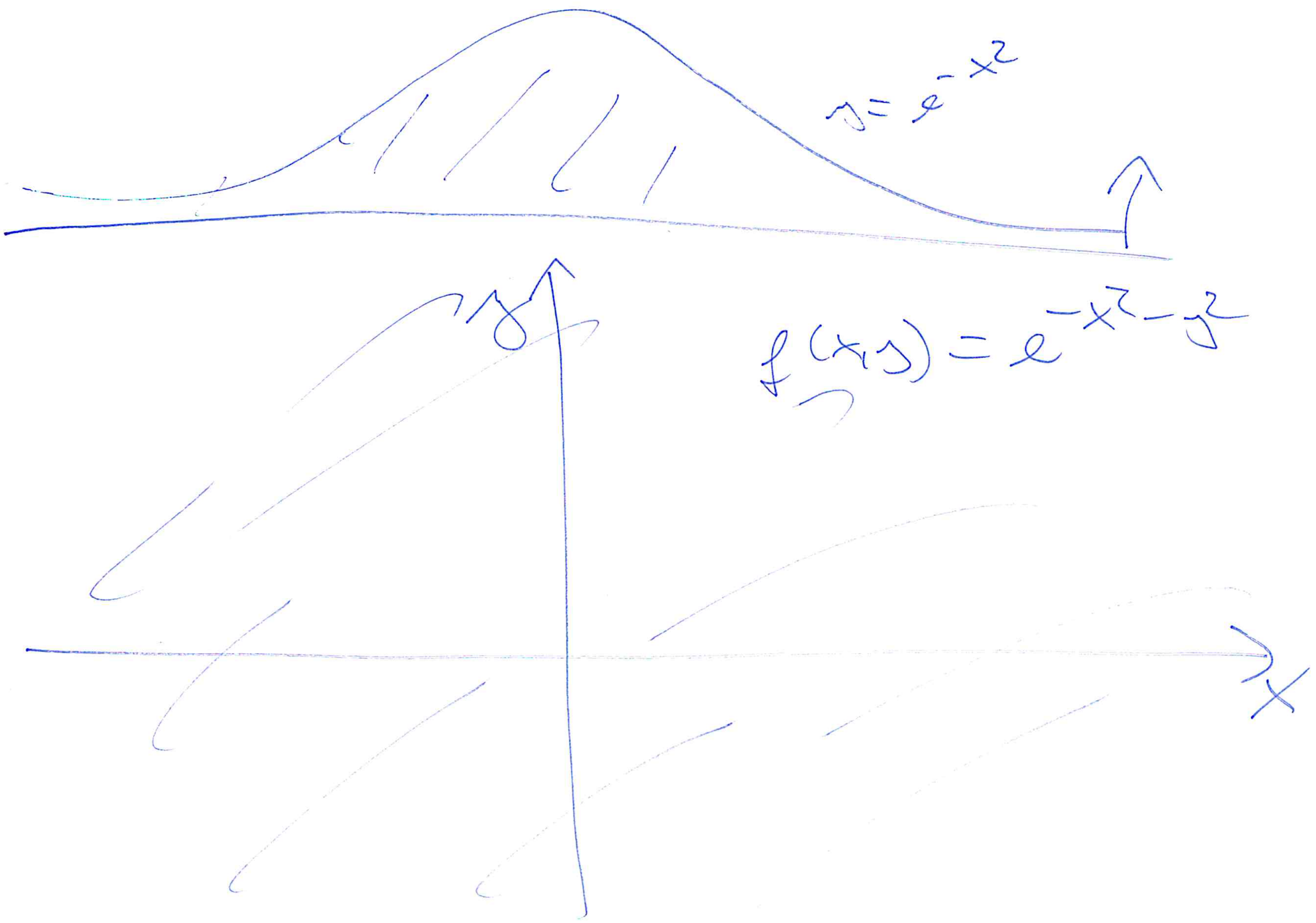
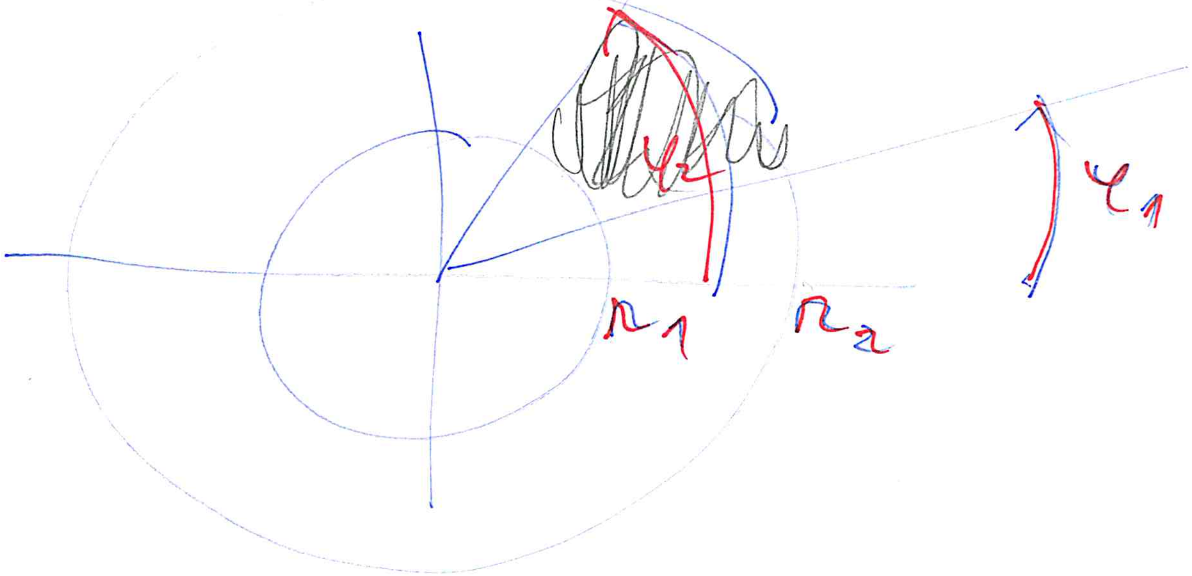

$y = e^{-x^2}$
$f(x,y) = e^{-x^2-y^2}$
y
x

$$I = \int_{\mathbb{R}} e^{-x^2} dx$$

$$I^2 = \iint_{\mathbb{R}^2} e^{-x^2-y^2} dx dy = \int_0^{2\pi} \left(\int_0^{+\infty} e^{-r^2} r dr \right) d\varphi$$

$\varphi \in [\varphi_1, \varphi_2]$ $r \in [r_1, r_2]$



$$\int_0^{+\infty} e^{-r^2} r\, dr = \left[-\frac{1}{2} e^{-r^2} \right]_0^{+\infty} = \lim_{r \to +\infty} \left(-\frac{1}{2} e^{-r^2} \right) - \lim_{r \to 0^+} \left(-\frac{1}{2} e^{-r^2} \right)$$

$$t = -r^2$$

$$= 0 - \left(-\frac{1}{2} \right) = \frac{1}{2}$$

$$\int_0^{2\pi} \frac{1}{2}\, d\varphi = \left[\frac{1}{2} \varphi \right]_0^{2\pi} = \pi$$

$$I = \int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$