$$\int \frac{1}{x^4+1}\,dx$$

$$\frac{1}{x^4+1} = \frac{\quad}{\quad} + \frac{\quad}{\quad}$$

$$x^4+1 = (x^2+x\sqrt{2}+1)(x^2-x\sqrt{2}+1)$$

$$x^4+1 = (x^2+1)^2 - (x\sqrt{2})^2$$

$$A^2-B^2=(A-B)(A+B)$$

$$x^2+x\sqrt{2}+1 \qquad\qquad x^2-x\sqrt{2}+1$$

$$x_{1,2}=\frac{-\sqrt{2}\pm i\sqrt{2}}{2} = \qquad\qquad x_{1,2} = \frac{\sqrt{2}}{2}(1\pm i)$$

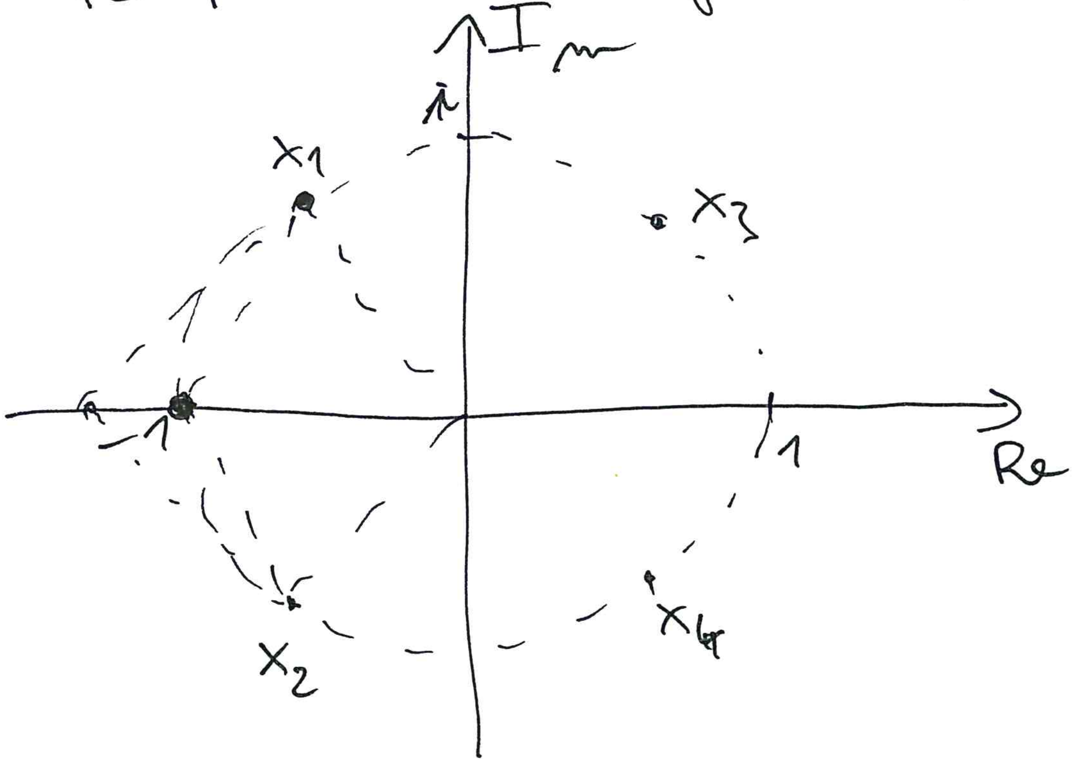
$$= \frac{\sqrt{2}}{2}(-1\pm i) \qquad |x_1|=1 \quad |x_2|=1 \qquad |x_3|=1 \quad |x_4|=1 \qquad \left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$$

$$x^3-1=(x-1)(\qquad)$$

$$x^2+1$$

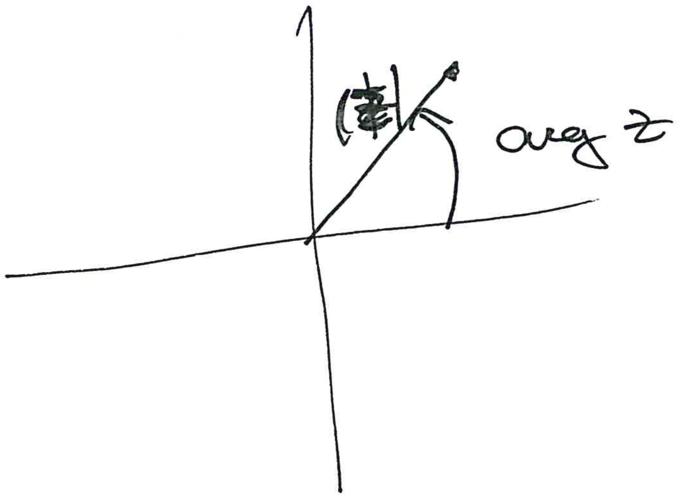
$$x^2-1=(x-1)(x+1)$$

Komplexní rovina (Gaussova)



$z = x + iy$

$|z| = \sqrt{x^2+y^2}$



kořeny $z^n = a$ tvoří vrcholy pravidelného n-úhelníku

Jak z $x_1, \dots, x_4$ dostat rozklad $x^4+1 = (x^2+x\sqrt{2}+1)(x^2 \dots)$

$x_1 \cdot x_2 = 1$ $x_3 \cdot x_4 = 1$

$|x_1 \cdot x_2| = 1$

Víme: x^4+1, x_1, x_2, x_3, x_4

$(x^4+1) : (x - x_1) =$

$$\frac{1}{x^4+1} = \frac{\cancel{1}\ Ax+B}{x^2+x\sqrt{2}+1} + \frac{\cancel{1}\ Cx+D}{x^2-x\sqrt{2}+1}$$

$$(x - x_1)(x - x_2) = x^2 - x(x_1 + x_2) + x_1 \cdot x_2$$

$$x^2 - x\sqrt{2} + 1 \qquad = 1$$

$$(x - x_3)(x - x_4) = x^2 + x\sqrt{2} + 1$$

$$\frac{1}{x^4+1} = \frac{Ax+B}{x^2+x\sqrt{2}+1} + \frac{Cx+D}{x^2-x\sqrt{2}+1}$$

$$1 = (Ax+B)(x^2-x\sqrt{2}+1) + (Cx+D)(x^2+x\sqrt{2}+1)$$

$$x^3: \ 0 = A + C$$

$$x^2: \ 0 = -\sqrt{2}A + B + C\sqrt{2} + D$$

$$x^1: \ 0 = A - \sqrt{2}B + C + D\sqrt{2}$$

$$x^0: \ 1 = B + D$$

$$B = D = \frac{1}{2} \qquad A = \frac{\sqrt{2}}{4}$$

$$C = -\frac{\sqrt{2}}{4}$$

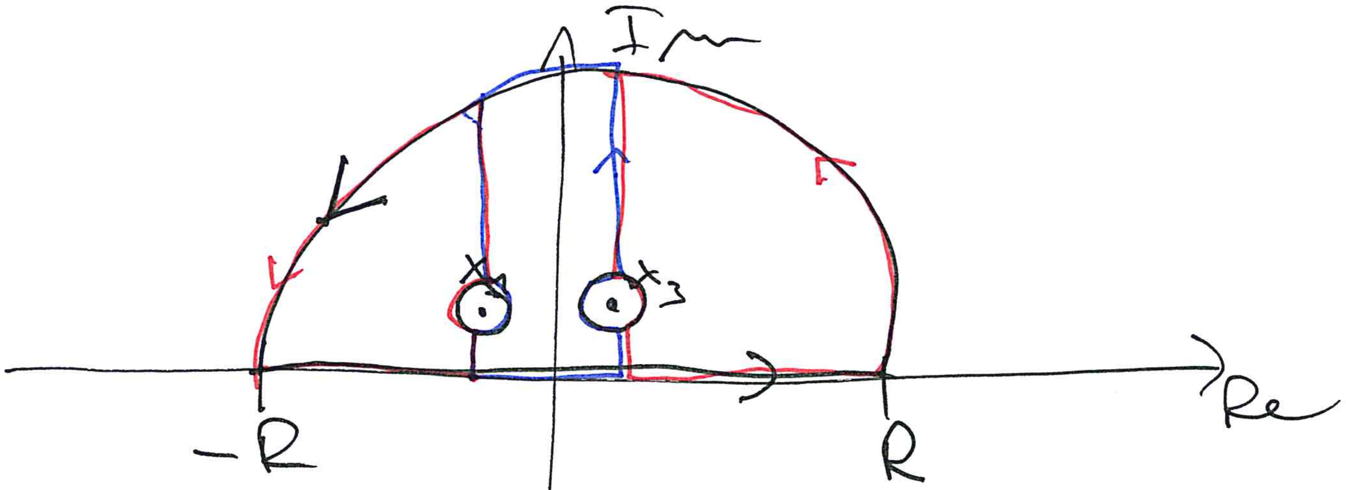
$$A + C = 0 \qquad C = -A$$

$$-\sqrt{2}A + C\sqrt{2} + 1 = 0$$

$$-2\sqrt{2}A + 1 = 0$$

$$A = \frac{1}{2\sqrt{2}}$$

$$\int_{\mathbb{R}} \frac{1}{x^4+1}\,dx = \int \frac{\frac{\sqrt{2}}{4}x + \frac{1}{2}}{x^2 + x\sqrt{2} + 1}\,dx + \int \frac{-\frac{\sqrt{2}}{4}x + \frac{1}{2}}{x^2 - x\sqrt{2} + 1}\,dx$$



$$\oint \frac{1}{z^4+1}\,dz = \int_{-R}^{R} \frac{1}{x^4+1}\,dX + \int_{\curvearrowleft} \frac{1}{z^4+1}\,dz$$

$R \to \infty$

$$\downarrow \qquad\qquad \downarrow$$

$$\int_{-\infty}^{+\infty} \frac{1}{x^4+1}\,dX \qquad\qquad 0$$

$$\int_{x_1} \qquad + \int$$

$$\frac{\dfrac{z-x_1}{z^4+1}}{z-x_1}$$

$$= 2\pi i\left(-\frac{x_1}{4} - \frac{x_3}{4}\right) =$$

$$= \frac{-2\pi i}{4}\left(x_1 + x_3\right) =$$

$$= \frac{-2\pi i}{4}\sqrt{2}\, i = \boxed{\frac{2\pi\sqrt{2}}{4}} = \frac{\pi\sqrt{2}}{2}$$

$$\lim_{z\to x_1} \frac{z-x_1}{z^4+1} \overset{L'H}{=} \lim_{z\to x_1} \frac{1}{4z^3} = \lim_{z\to x_1} \frac{z}{4z^4} = \frac{x_1}{4x_1^4} = -\frac{x_1}{4}$$