

$$5. \quad \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$12. \quad |z_1 - z_2| \leq |z_1| + |z_2|$$

$$5. \quad z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

$$L = \overline{z_1 + z_2} = \overline{x_1 + x_2 + i(y_1 + y_2)} = x_1 + x_2 - i(y_1 + y_2)$$

$$P = \overline{z_1} + \overline{z_2} = \overline{x_1 + iy_1} + \overline{x_2 + iy_2} = x_1 - iy_1 + x_2 - iy_2 = \\ = x_1 + x_2 - i(y_1 + y_2)$$

$$L = P$$

$$12. \quad z_1 - z_2 = x_1 - x_2 + i(y_1 - y_2)$$

$$L = |z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$P = \sqrt{x_1^2 + y_1^2} + \sqrt{x_2^2 + y_2^2}$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \leq \sqrt{x_1^2 + y_1^2} + \sqrt{x_2^2 + y_2^2} \quad (2)$$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 \leq x_1^2 + y_1^2 + x_2^2 + y_2^2 + 2\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}$$

$$A \leq B \Leftrightarrow A^2 \leq B^2 \quad \text{for } A, B \geq 0$$

$$\underline{x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2} \leq$$

$$|-2x_1x_2 - 2y_1y_2| \leq 2\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2} \quad |^2$$

$$(-x_1x_2 - y_1y_2)^2 \leq (x_1^2 + y_1^2)(x_2^2 + y_2^2)$$

$$\underline{x_1^2x_2^2 + 2x_1x_2y_1y_2 + y_1^2y_2^2} \leq \underline{x_1^2x_2^2 + y_1^2x_2^2 + x_1^2y_2^2 + y_1^2y_2^2}$$

$$0 \leq \cancel{y_1^2x_2^2} - 2x_1x_2y_1y_2 + \cancel{y_1^2y_2^2} + \cancel{x_1^2x_2^2} + \cancel{x_1^2y_2^2}$$

$$0 \leq (y_1x_2 - y_2x_1)^2 \quad \text{flat}$$