$$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

$$\lim_{x \to x_0} f(x) = L \quad .. \quad (\forall \varepsilon > 0)(\exists \delta > 0)(\forall x \in P_\delta(x_0))(f(x) \in U_\varepsilon(L))$$

$$\cancel{P_\delta(x_0) = (x_0 - \delta, x_0) \cup (x_0, x_0 + \delta)}$$

$$P_\delta(x_0) = \{x : 0 < |x - x_0| < \delta\}$$

$$\cancel{U_\varepsilon(L) = (L - \varepsilon, L + \varepsilon)}$$

$$U_\varepsilon(L) = \{y : |y - L| < \varepsilon\}$$

$$f(z) = z^2 \qquad z = x + iy$$

$$f(z) = x^2 + 2xyi - y^2$$

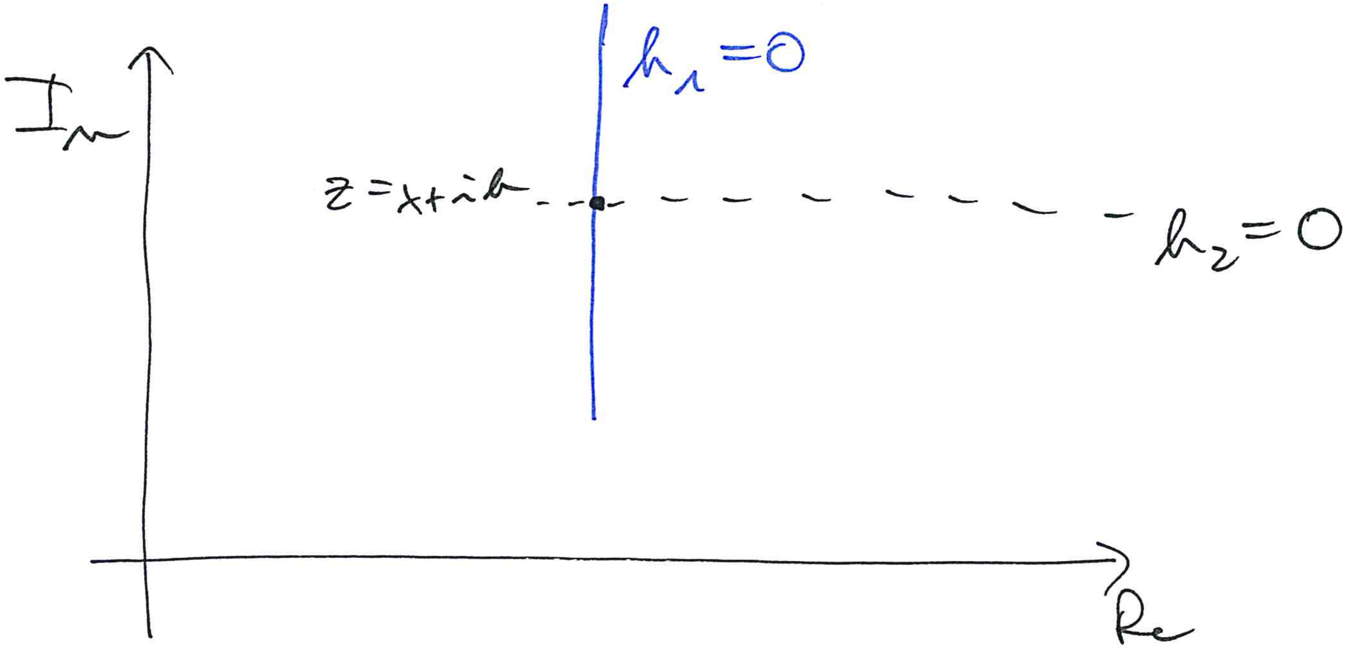
$$f_1(x,y) = x^2 - y^2 \qquad f_1(z) = x^2 - y^2$$

$$f_2(x,y) = 2xy \qquad f_2(z) = 2xy$$

$$h = h_1 + i h_2$$

$$\lim_{h \to 0} \frac{f(z+h) - f(z)}{h} = \lim_{(h_1, h_2) \to (0,0)} \frac{f_1(z+h) + i f_2(z+h) - f_1(z) - i f_2(z)}{h_1 + i h_2} =$$

$$= \lim_{(h_1, h_2) \to (0,0)} \frac{f_1(x + \cancel{h_1}, y + h_2) + i f_2(x + \cancel{h_1}, y + h_2) - f_1(x,y) - i f_2(x,y)}{\cancel{h_1} + i h_2}$$



$$\lim_{h_1 \to 0} \frac{f_1(x+h_1,y) + i\, f_2(x+h_1,y) - f_1(x,y) - i\, f_2(x,y)}{h_1} =$$

$$= \underbrace{\lim_{h_1 \to 0} \frac{f_1(x+h_1,y) - f_1(x,y)}{h_1}} + \lim_{h_1 \to 0} \frac{i\left(f_2(x+h_1,y) - f_2(x,y)\right)}{h_1}$$

$$\frac{\partial f_1}{\partial x} \quad + \quad i\, \frac{\partial f_2}{\partial x}$$

$$\frac{\partial f_1}{\partial x} + i\frac{\partial f_2}{\partial \underset{x}{\cancel{y}}} = \underline{2x + i2y} = 2(x+iy) = 2z$$

$$\frac{1}{i}\left(\frac{\partial f_1}{\partial y} + i\frac{\partial f_2}{\partial y}\right) = \underset{-i}{\frac{1}{i}}\frac{\partial f_1}{\partial y} + \frac{\partial f_2}{\partial y} =$$

$$= -i(-2y) + 2x = \underline{2x + 2iy}$$

Cauchy – Riemannovy podmínky

$$\frac{\partial f_1}{\partial x} + i\frac{\partial f_2}{\partial x} = \frac{\partial f_2}{\partial y} - i\frac{\partial f_1}{\partial y}$$

$$\frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial y} \quad \Big| \quad \frac{\partial f_2}{\partial x} = -\frac{\partial f_1}{\partial y}$$

$$f(z) = z^3 = (x+iy)^3 = x^3 + 3x^2 iy + 3x\, i^2 y^2 + (iy)^3$$

$$f_1(x,y) = x^3 - 3xy^2$$

$$f_2(x,y) = 3x^2 y - y^3$$

$$\frac{\partial f_1}{\partial x} = 3x^2 - 3y^2 \quad \frac{\partial f_1}{\partial y} = -6xy$$

$$\frac{\partial f_2}{\partial x} = 6xy \quad \frac{\partial f_2}{\partial y} = 3x^2 - 3y^2$$

$$f'(z) = \frac{\partial f_1}{\partial x} + i \frac{\partial f_2}{\partial x} = 3x^2 - 3y^2 + i\, 6xy =$$

$$= 3\left(\underbrace{x^2 - y^2 + 2ixy}_{(x+iy)^2} \right) = 3z^2$$

$$\cancel{\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \longmapsto \begin{pmatrix} f \\ \end{pmatrix}}$$

$$f(z) = z^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix} \qquad \begin{pmatrix} x^2 - y^2 \\ 2xy \end{pmatrix}$$

silná derivace funkce f_1 f_2 :

$$L : \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \longmapsto \frac{\partial f_1 \; f_2}{\partial x} \cdot h_1 + \frac{\partial f_1 \; f_2}{\partial y} h_2$$

silná derivace funkce $f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \longmapsto \begin{pmatrix} \frac{\partial f_1}{\partial x} h_1 + \frac{\partial f_1}{\partial y} h_2 \\ \frac{\partial f_2}{\partial x} h_1 + \frac{\partial f_2}{\partial y} h_2 \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial x} , & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} , & \frac{\partial f_2}{\partial y} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$1 + 0 \cdot i \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} \sim$

$\sim a + ic$

$0 + 1 \cdot i \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} \sim$

$\sim b + id$

v $\mathbb{R}^2$ jsou $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ LNZ

v $\mathbb{C}$ jsou $1, i$ LZ $\qquad i = i \cdot 1$

odtud se dají odvodit C-R podmínky