Definice:

Nechť $\Omega \subseteq \mathbb{C}$ je otevřená množina, f je funkce $\Omega \to \mathbb{C}$.
Řekneme, že je f holomorfní na Ω, pokud pro $z \in \Omega$ existuje $f'(z)$.

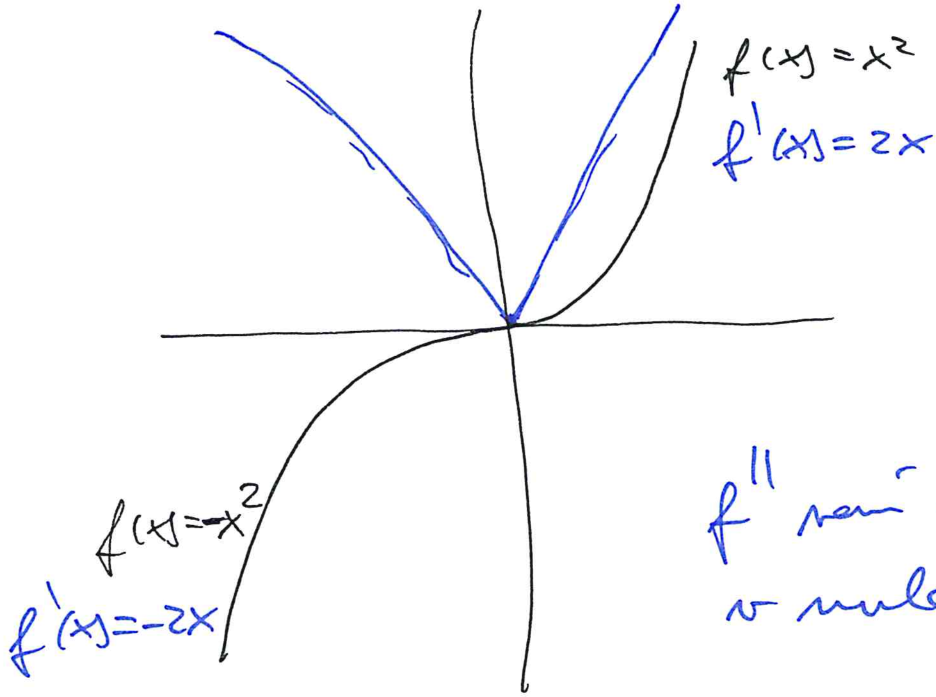
Příklady: $f(z) = z^2$ je holomorfní na $\mathbb{C}$

$f(z) = \frac{1}{z}$ je — " — na $\mathbb{C} \setminus \{0\}$

C-R podmínky: $f(z) = f_1(x,y) + i f_2(x,y)$, $z = x + iy$

$$\frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial y}, \quad \frac{\partial f_1}{\partial y} = -\frac{\partial f_2}{\partial x}$$

Reálný obor: $f(x) = x \cdot |x|$



f'' není definovaná v nule

$$f'(x) = 2|x|$$

V $\mathbb{C}$ platí: Je-li f holomorfní na množině M, pak je f' holomorfní na M.

f holomorfní na M

∂f_1

$f(z) = f_1(x,y) + i f_2(x,y)$

$$f'(z) = \frac{\partial f_1}{\partial x} + i \frac{\partial f_2}{\partial x}$$

$$\frac{\partial f_2}{\partial y} - i \frac{\partial f_1}{\partial y}$$

C-R podmínky pro f'

$$\frac{\partial}{\partial x}\left(\frac{\partial f_1}{\partial x}\right) = \frac{\partial}{\partial y}\left(-\frac{\partial f_1}{\partial y}\right)$$

$$\frac{\partial^2 f_1}{\partial x^2} + \frac{\partial^2 f_1}{\partial y^2} = 0$$

Definice: Nechť $M \subseteq \mathbb{R}^2$ je otevřená množina, $f: M \to R$.
Funkci f nazveme harmonickou na M, pokud na M platí

$$\underbrace{\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}}_{\Delta f} = 0$$

Jiné odvození:

$$\left.\begin{aligned} \frac{\partial f_1}{\partial x} &= \frac{\partial f_2}{\partial y} \quad \Big| \frac{\partial}{\partial x} \\ \frac{\partial f_1}{\partial y} &= -\frac{\partial f_2}{\partial x} \quad \Big| \frac{\partial}{\partial y} \end{aligned}\right\} +$$

$$\frac{\partial^2 f_1}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \qquad \left(\frac{\partial}{\partial x}\left(\frac{\partial f_2}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f_2}{\partial x}\right)\right)$$

$$f(z) = z^4 = (x+iy)^4 =$$

$$= x^4 + 4i\,x^3 y - 6\,x^2 y^2 - 4i\,x y^3 + y^4$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$f_1(x,y) = x^4 - 6x^2y^2 + y^4$$

$$f_2(x,y) = 4x^3y - 4xy^3$$

$$\left.\begin{array}{l} \dfrac{\partial^2 f_1}{\partial x^2} = 12x^2 - 12y^2 \\[2ex] \dfrac{\partial^2 f_1}{\partial y^2} = -12x^2 + 12y^2 \end{array}\right\} + \quad = 0$$

$$\left.\begin{array}{l} \dfrac{\partial^2 f_2}{\partial x^2} = 4\cdot 3\cdot 2\; xy \\[2ex] \dfrac{\partial^2 f_2}{\partial y^2} = -4x\cdot 3\cdot 2\,y \end{array}\right\} + \quad = 0$$