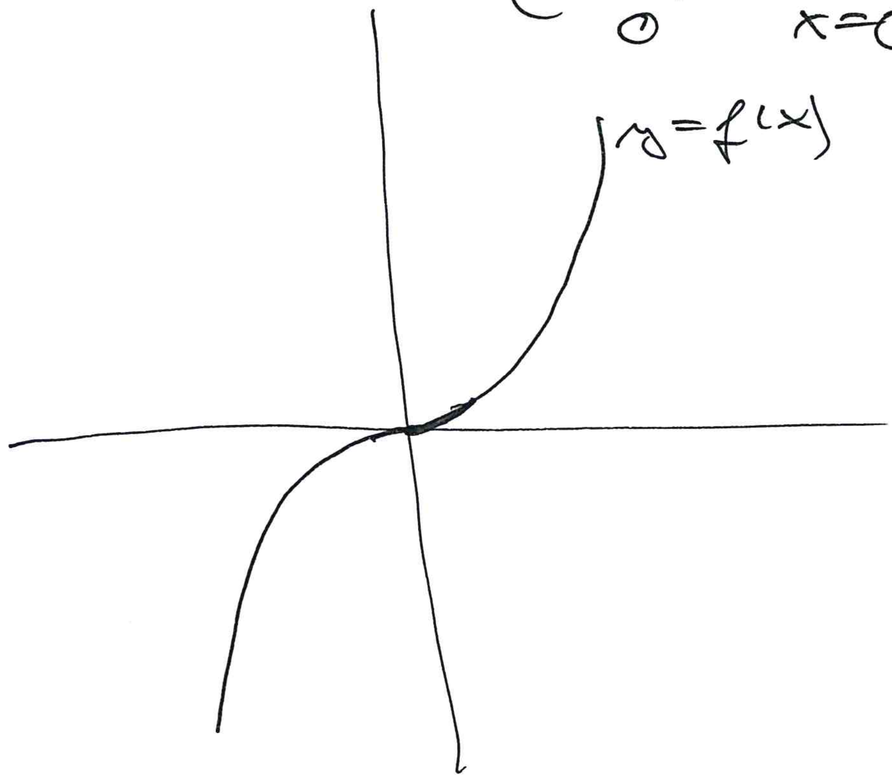
$$\left.\begin{aligned} f(z) &= z|z| \\ g(z) &= \exp\left(-\frac{1}{z^2}\right) \end{aligned}\right\} \quad \begin{aligned} &\text{v reálném oboru} \\ &\text{v komplexním oboru} \end{aligned}$$

f má na $\mathbb{R}$ derivaci f'
ale v $x=0$ nemá f''

$$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x \leq 0 \end{cases}$$

$$f'(x) = \left.\begin{cases} 2x & x > 0 \\ -2x & x < 0 \\ 0 & x = 0 \end{cases}\right\} = 2|x|$$



$f(z) = z|z|$ v komplexním oboru

$z = x + iy$

$$f(z) = (x+iy)\sqrt{x^2+y^2} = x\sqrt{x^2+y^2} + i\,y\sqrt{x^2+y^2}$$

$$f_1(x,y) = x\sqrt{x^2+y^2}$$

$$f_2(x,y) = y\sqrt{x^2+y^2}$$

C-R podmínky:

$$\frac{\partial f_1}{\partial x} = \sqrt{x^2+y^2} + x \cdot \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x = \sqrt{x^2+y^2} + \frac{x^2}{\sqrt{x^2+y^2}} =$$

$$= \frac{2x^2+y^2}{\sqrt{x^2+y^2}}$$

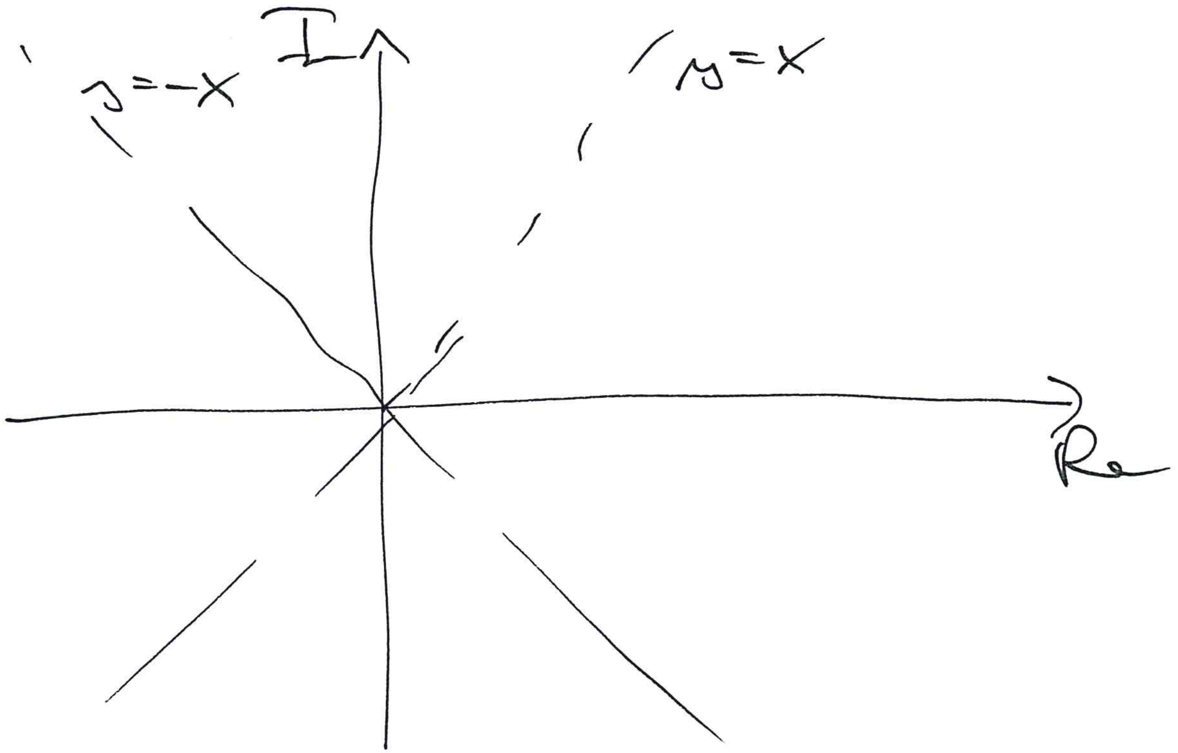
$$\frac{\partial f_2}{\partial y} = \sqrt{x^2+y^2} + y\,\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y = \sqrt{x^2+y^2} + \frac{y^2}{\sqrt{x^2+y^2}} =$$

$$\frac{\partial f_1}{\partial x} \neq \frac{\partial f_2}{\partial y} \qquad = \frac{x^2+2y^2}{\sqrt{x^2+y^2}}$$

Pro jaká x, y je $\dfrac{\partial f_1}{\partial x} = \dfrac{\partial f_2}{\partial y}$?

$$\frac{2x^2+y^2}{\sqrt{x^2+y^2}} = \frac{x^2+2y^2}{\sqrt{x^2+y^2}} \quad / \cdot \sqrt{x^2+y^2} \qquad -x^2-y^2$$

$$x^2 = y^2$$



Závěr:

Na $\mathbb{R}$ má f derivaci f'

Na $\mathbb{C}$ ~~jsou~~ je splněna C-R podmínka jen ve dvou přímkách, tedy na $\mathbb{C}$ nemá f derivaci ani v bodech reálné osy

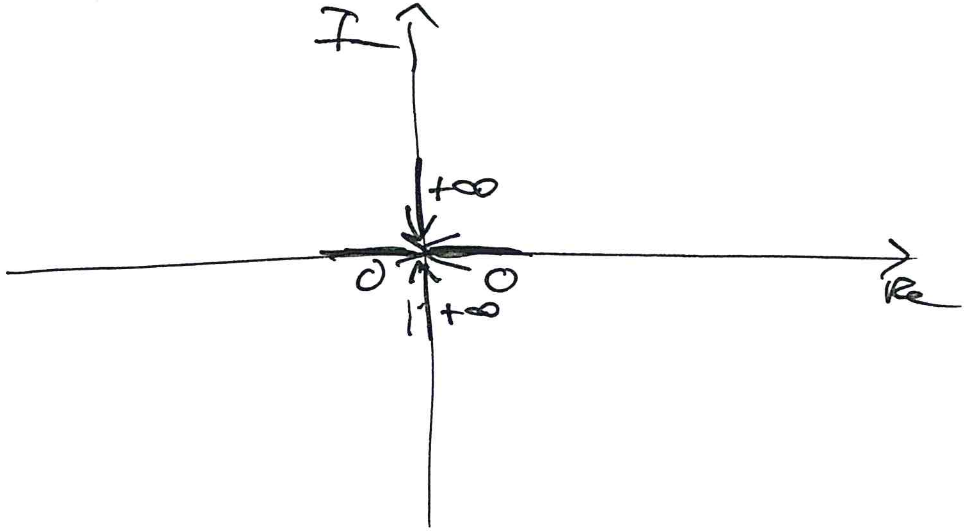
$g(z) = \exp\left(-\frac{1}{z^2}\right)$ ~ v $z=0$ není definovaná

má derivaci na $\mathbb{R} \setminus \{0\}$

$\mathbb{C} \setminus \{0\}$

$\lim\limits_{z \to 0} g(z)$ v reálném × komplexním oboru

$\to 0$



v reálném oboru

Spojité rozšíření:

$$\tilde{g} : x \mapsto \begin{cases} g(x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$\tilde{g}'(0) = 0$

$(\forall n \in \mathbb{N})\left(\tilde{g}^{(n)}(0) = 0\right)$

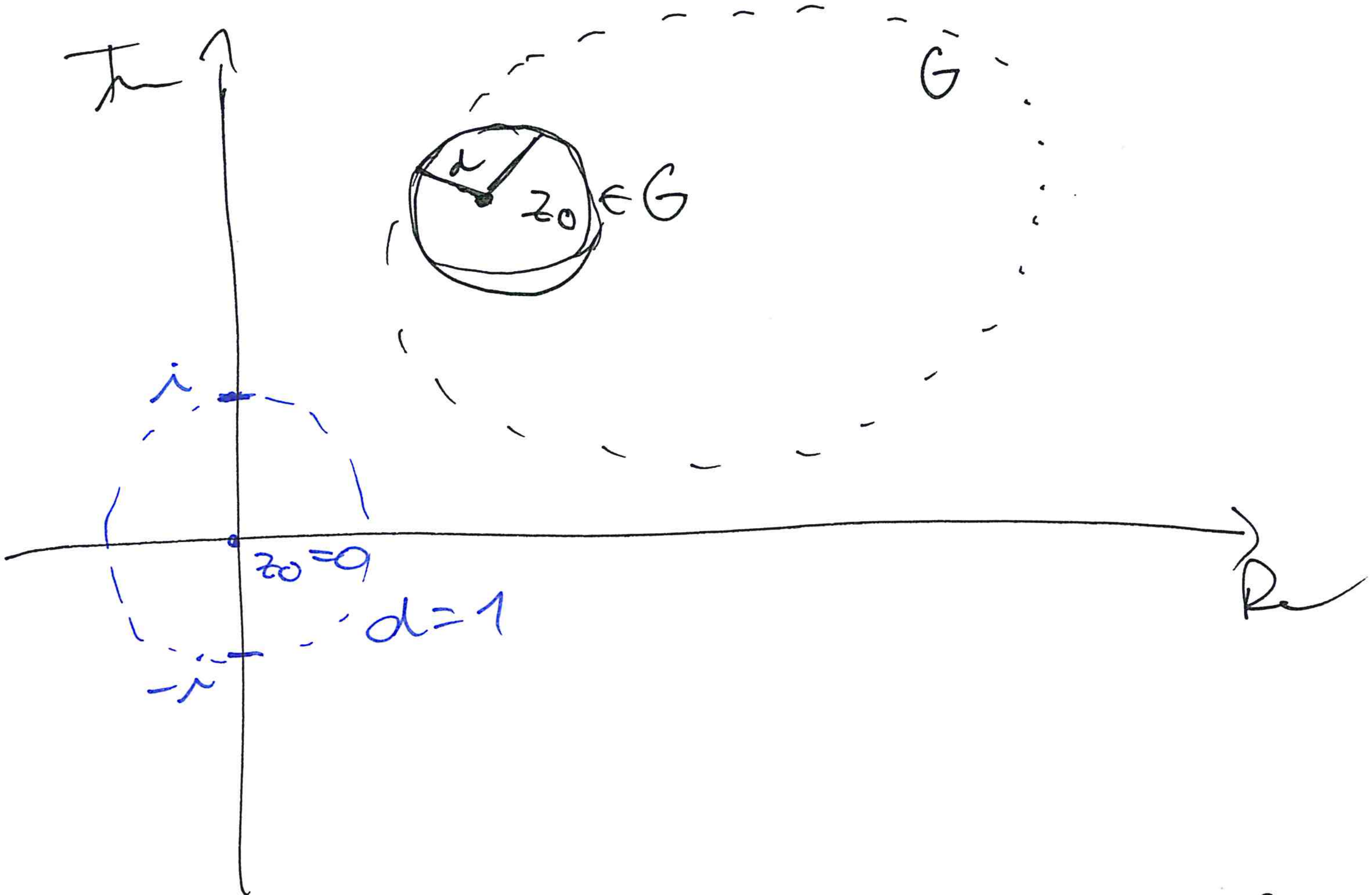
T. Ř. v nule je $T(x) = \sum 0 \cdot x^n = 0$

zřejmě $T(x) \neq \tilde{g}(x)$ pro $x \neq 0$

$z = iy$

$-\frac{1}{z^2} = -\frac{1}{(iy)^2} = \frac{1}{y^2}$

nemá limitu



Příklad: $f(z) = \dfrac{1}{1+z^2} = 1 - z^2 + z^4 - z^6 + \dots$

$g = -z^2$

konverguje na

$M = \{ z \in \mathbb{C} : |z| < 1 \}$

$$G = \mathbb{C} \setminus \{ i, -i \}$$

Závěr plynoucí z věty 5.9.8:

V komplexním oboru nemůže nastat případ funkce $\tilde{g}$ z reálného oboru.

C–R pogoj $\qquad f(z) = \dfrac{1}{1+z^2}$

prav je $(\forall z_1, z_2 \in \mathbb{C})\big(\exp(z_1 + z_2) = \exp(z_1) \cdot \exp(z_2)\big)$

$$e^{x+iy} = e^{x} \cdot e^{iy}$$

$f_1(x,y) = ?$ $\quad$ $f_2(x,y) = ?$

$$1+z^2 = 1+(x+iy)^2 = 1+x^2-y^2+2ixy$$

$$\frac{1}{1+x^2-y^2+2ixy} = \frac{1+x^2-y^2-2ixy}{(1+x^2-y^2)^2+4x^2y^2}$$

$$\boxed{\frac{1}{w} = \frac{\overline{w}}{|w|^2}}$$

$$f_1(x,y) = \frac{1+x^2-y^2}{(1+x^2-y^2)^2+4x^2y^2}$$

$$f_2(x,y) = \frac{-2xy}{(1+x^2-y^2)^2+4x^2y^2}$$

$$\frac{\partial f_1}{\partial x} = \frac{2x\left[(1+x^2-y^2)^2+4x^2y^2\right] - \left[2(1+x^2-y^2)\cdot 2x + 8xy^2\right]\cdot(1+x^2-y^2)}{\left[\qquad\right]^2}$$

$$\frac{\partial f_2}{\partial y} = \frac{-2x\left[(1+x^2-y^2)^2+4x^2y^2\right] + 2xy\left[2(1+x^2-y^2)(-2y)+8x^2y\right]}{\left[\qquad\right]^2}$$

$$8x^3y^2 = -8x^3y^2 + 16x^3y^2$$

Závěr $\frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial y}$

Chcete počítat? $\frac{\partial f_1}{\partial y} = -\frac{\partial f_2}{\partial x}$?

$$\frac{\partial f_1}{\partial y} = \frac{-2y\left[(1+x^2-y^2)^2 + 4x^2y^2\right] - (1+x^2-y^2)\left[2(1+x^2-y^2)(-2y) + 8x^2y\right]}{\left[\qquad\qquad\right]^2}$$

$$= \frac{(1+x^2-y^2)^2(-2y+4y) - 8x^2y(1+x^2-y^2) - 8x^2y^3}{\left[\qquad\qquad\right]^2}$$

$$\frac{\partial f_2}{\partial x} = \frac{-2y\left[(1+x^2-y^2)^2 + 4x^2y^2\right] + 2xy\left[2(1+x^2-y^2)\,2x + 8xy^2\right]}{\left[\qquad\qquad\right]^2}$$

$$= \frac{-2y(1+x^2-y^2)^2 - 8x^2y^3 + 8x^2y(1+x^2-y^2) + 16x^2y^3}{\left[\qquad\qquad\right]^2}$$