$$f(z) = \operatorname{erf}(2z) - (\operatorname{erf}(z))^2 = \sum_{k=0}^{\infty} a_k z^k$$

$$a_k z^k + a_{k+1} z^{k+1} + \dots$$

$$(n=k \qquad n=k+1)$$

$$= z^k \left( a_k + a_{k+1} z + a_{k+2} z^2 + \dots \right)$$

$$a_k \neq 0$$

---

ze spojitosti plyne, že $\exists\, \delta > 0$:

v okolí $z=0$ na reálné ose ; $(0-\delta, 0+\delta)$

je jediný kořen $z=0$ $\Downarrow$ není pravé

proto $a_n = 0$ pro všechna $n \in \mathbb{N}$ a tedy

$f(z) = 0$

$e^{z_1+z_2} = e^{z_1} \cdot e^{z_2}$ pro $z_1, z_2 \in \mathbb{C}$

víme: pro $z_1, z_2 \in \mathbb{R}$ to platí

$$f(z) = e^{z_1+z} - e^{z_1} \cdot e^{z}, \quad z_1 \in \mathbb{R}$$

$(\forall z \in \mathbb{R})(f(z) = 0)$

stejnou úvahou jako v minulém příkladě ukážeme, že

$f(z) = 0$ pro $z \in \mathbb{C}$

tedy víme: $z_2 \in \mathbb{C}$, $z_1 \in \mathbb{R}$, tak $\exp(z_1 + z_2) = \exp(z_1) \cdot \exp(z_2)$

$$g(z) = \exp(z_1 + z) - \exp(z_1) \cdot \exp(z) \quad z_1 \in \mathbb{C}$$

víme, že pro $z \in \mathbb{R}$ je $g(z) = 0$

stejnou úvahou - - - - - - $g(z) = 0$ pro $z \in \mathbb{C}$

odtud: $(\forall z_1, z_2 \in \mathbb{C})(\exp(z_1 + z_2) = \exp(z_1) \cdot \exp(z_2))$

$$\exp(z_1)\cdot\exp(z_2) = \left(1+z_1+\frac{z_1^2}{2!}+\dots\right)\left(1+z_2+\frac{z_2^2}{2!}+\frac{z_2^3}{3!}+\dots\right)$$

$$\left(\sum_{k=0}^{\infty}\frac{1}{k!}z_1^k\right)\left(\sum_{\ell=0}^{\infty}\frac{1}{\ell!}z_2^\ell\right)$$

$$\left(1+z_1+\frac{z_1^2}{2!}+\frac{z_1^3}{3!}+\dots\right)\left(1+z_2+\frac{z_2^2}{2!}+\frac{z_2^3}{3!}+\dots\right)$$

$1\cdot 1$

$1\cdot z_2$    $z_1\cdot 1$

$1\cdot\frac{z_2^2}{2!}$    $z_1\cdot z_2$    $\frac{z_1^2}{2!}\cdot 1$

$1\cdot\frac{z_2^3}{3!}$    $z_1\cdot\frac{z_2^2}{2!}$    $\frac{z_1^2}{2!}\cdot z_2$    $\frac{z_1^3}{3!}\cdot 1$

$1\cdot\frac{z_2^4}{4!}$    $z_1\cdot\frac{z_2^3}{3!}$    $\frac{z_1^2}{2!}\,\frac{z_2^2}{2!}$    $\frac{z_1^3}{3!}\,z_2$    $\frac{z_1^4}{4!}\,1$

$$1+(z_1+z_2)+\frac{1}{2!}(z_1+z_2)^2+\frac{1}{3!}(z_1+z_2)^3+\frac{1}{4!}(z_1+z_2)^4+\dots$$

$\exp(z_1+z_2)$

Řešme v $\mathbb{C}$ rovnici $\exp(z) = 2-4i$

všimněme si: $\exp(z+2\pi i) = \exp(z)$

$$\exp(2\pi i) = \cos(2\pi) + i\sin(2\pi) = 1$$

závěr: má-li rovnice kořen $z_0$ v $\mathbb{C}$, pak jich má nekonečně mnoho

$$z_0 \quad \ldots \quad z_0+2\pi i, \quad z_0+4\pi i, \quad z_0-2\pi i \ldots$$

$z = x + iy$, $x = \mathrm{Re}(z)$, $y = \mathrm{Im}(z)$

$$\exp(z) = \exp(x)\cdot(\cos(y) + i\sin(y)) = 2-4i$$

abs. hodnota: $|\exp(x)|\cdot\underbrace{|(\cos(y)+i\sin(y))|}_{=1} = |2-4i|$

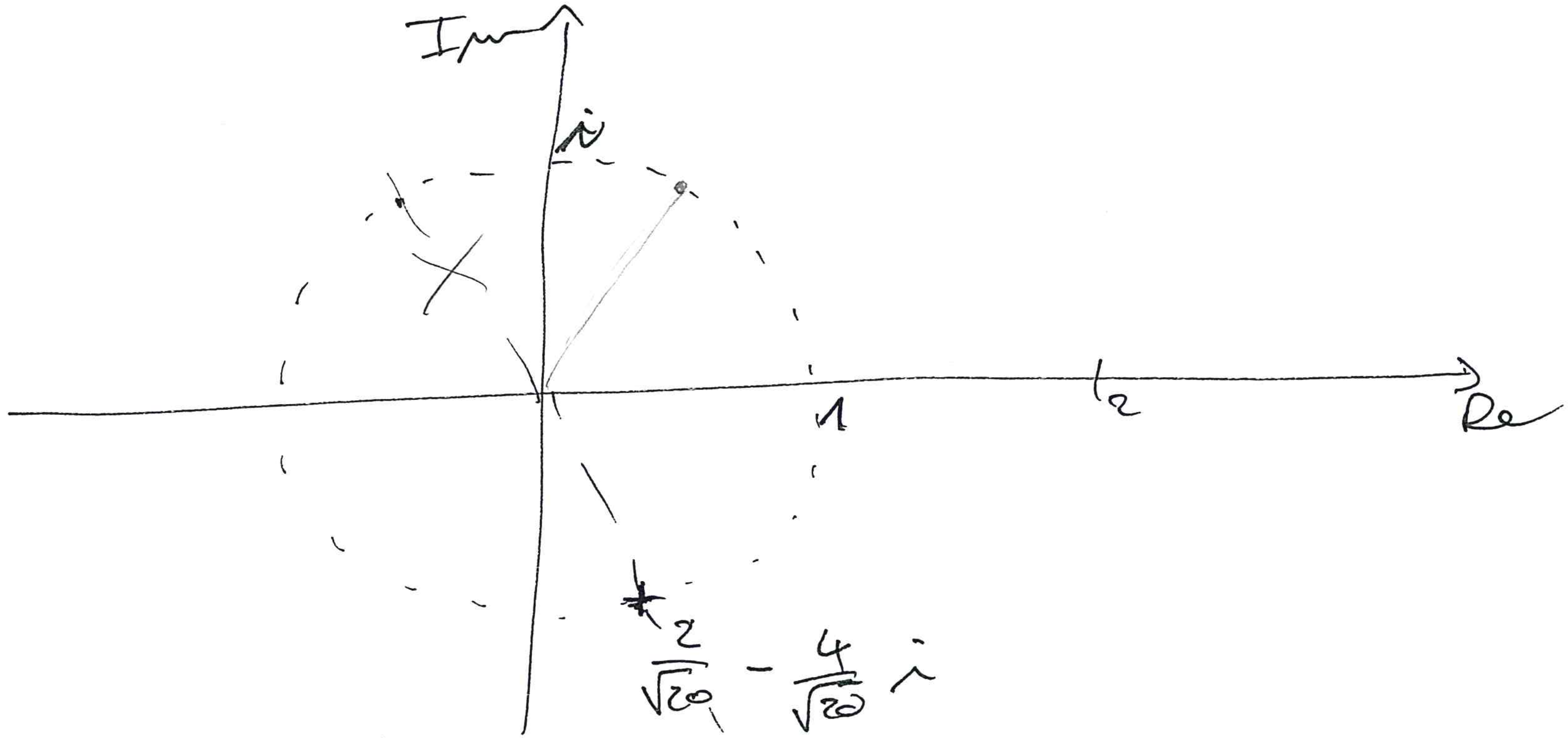
$$\exp(x) = \sqrt{20}$$

rovnice v reálném oboru:

$$x = \log\sqrt{20}$$

$$\sqrt{20}\left(\cos(y) + i\sin(y)\right) = 2 - 4i$$

$$\cos(y) + i\sin(y) = \frac{2}{\sqrt{20}} - \frac{4}{\sqrt{20}}i$$



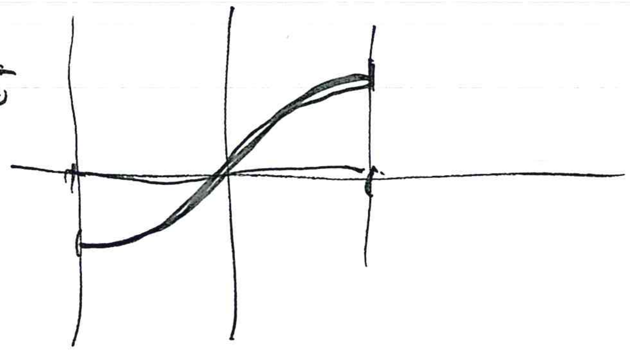

$$y = \arccos\frac{2}{\sqrt{20}} \quad ?$$

$$\sin y = -\frac{4}{\sqrt{20}}$$

$2-4i$

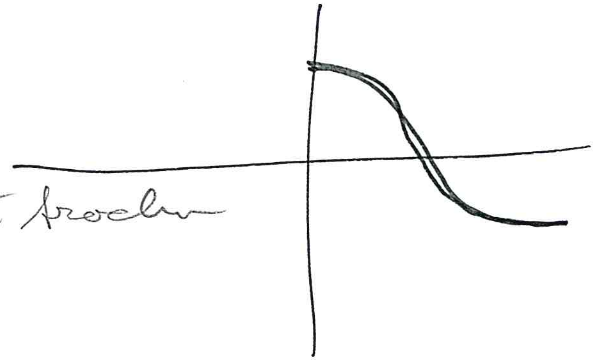
$$y = \arcsin\left(\frac{-4}{\sqrt{20}}\right) = -\arcsin\frac{4}{\sqrt{20}}$$

ano



$$y = -\arccos\frac{2}{\sqrt{20}} + \cancel{\pi}$$

je to ještě trochu jinak

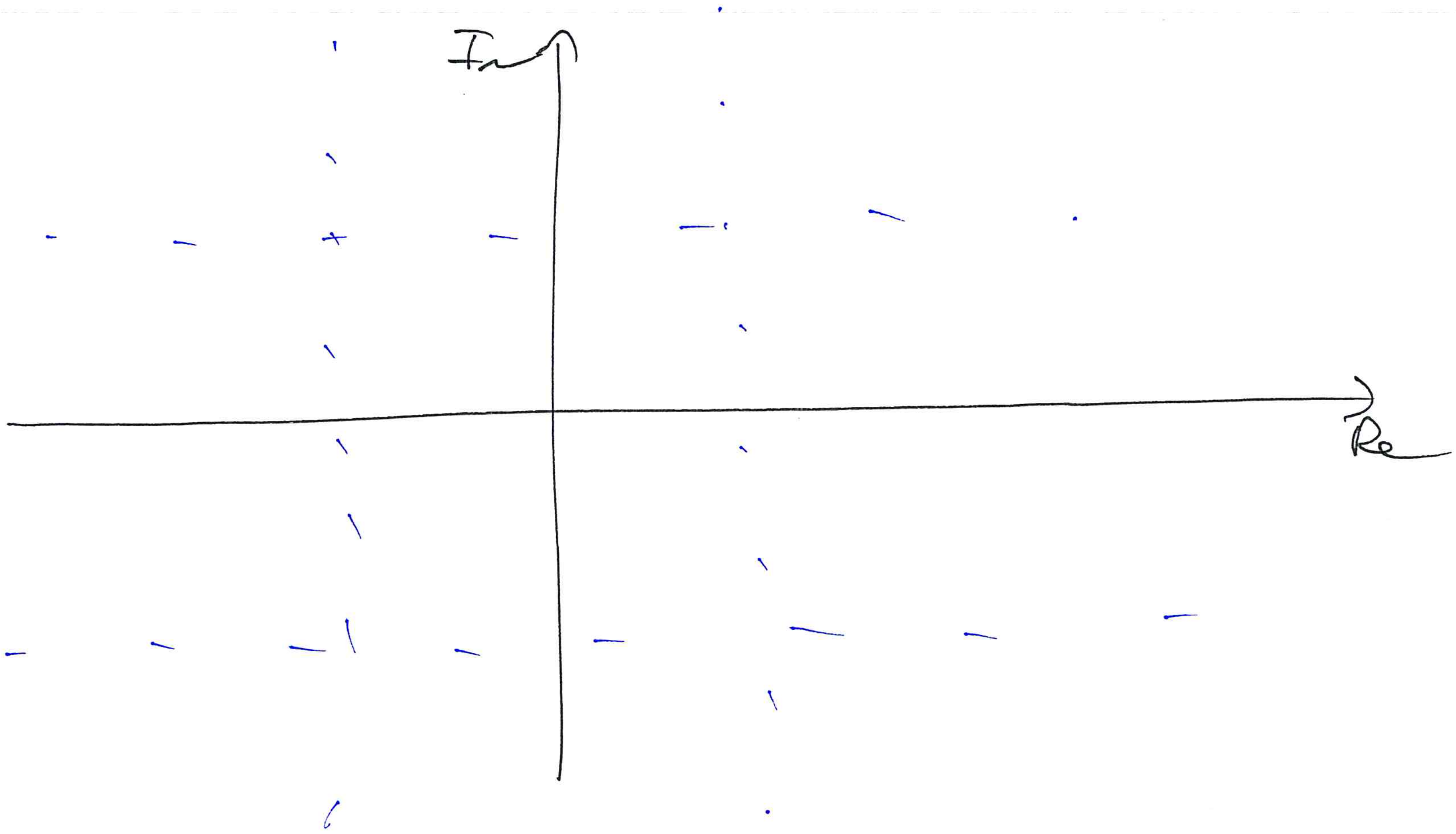


$$y = -\arcsin\frac{2}{\sqrt{5}}$$

závěr:

$$z_k = \log\sqrt{20} + i\left(-\arcsin\frac{2}{\sqrt{5}} + 2k\pi\right), \quad k \in \mathbb{Z}$$

jsou kořeny rovnice $\exp(z) = 2-4i$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