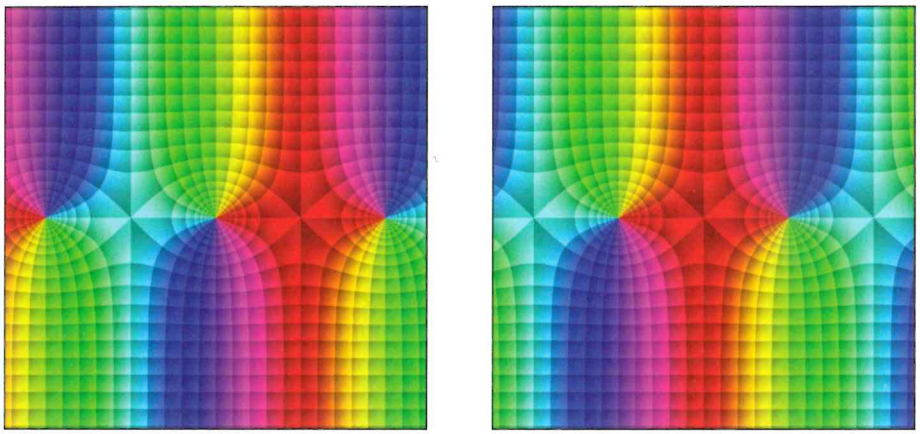

Figure 3.15: Enhanced phase portraits of the complex sine and cosine function

The most striking observation is that, apart from a translation, the phase portraits seem to be identical, though the defining series of the functions are totally different. Another remarkable fact is their invariance with respect to a horizontal shift, which becomes visible more clearly when the domain depicted is enlarged, as we did for the cosine function in Figure 3.16 (left). Furthermore we notice that at some distance off the real axis the phase portrait is composed of parallel vertical stripes. And here is a first rule of thumb in our training on phase portraits:

Whenever the phase portrait shows (almost) parallel stripes, the exponential function might be involved.

Indeed the stripes in the phase portraits are crucial for understanding the sine and the cosine functions. Since the deductions are slightly simpler for the cosine function, we work with it and leave the sine function as an exercise for the reader.

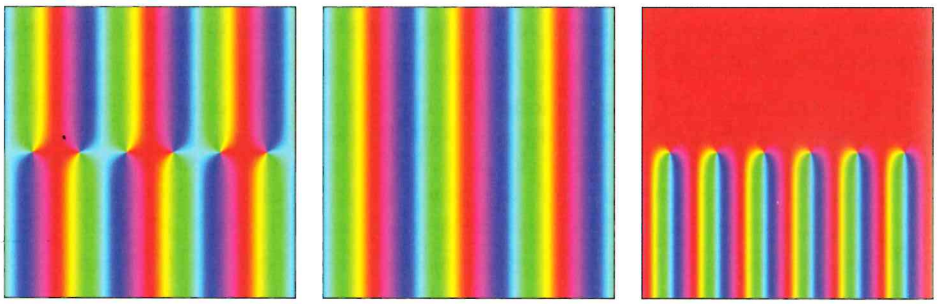

Figure 3.16: Phase portraits of $f(z) = \cos z$, $g(z) = \mathrm{e}^{-\mathrm{i}\,z}$, and $f(z)/g(z)$

Because the stripes in the phase portrait of the cosine function are vertical, we transform the exponential function appropriately, pre-composing it with the map-