Pädagogische Hochschule
Schwäbisch Gmünd
University of Education




MATHEMATIKUS

Software-supported development of spatial-visual abilities

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www.mathematikus.de

NORD
University

Overview

1 Importance of spatial-visual ability

2 Spatial-visual ability and its components – illustrated with tasks

3 Software to develop spatial ability

4 Mathematikus.de

5 open questions - research activities

1 What would we be without imaginations?

Lifelong: Every day we imagine something

- Wolves, princes and so on in fairy tales,
- Aliens,
- expectations (and fulfillment?),
- Plans of beautiful things,
- Inventions of things that never existed...

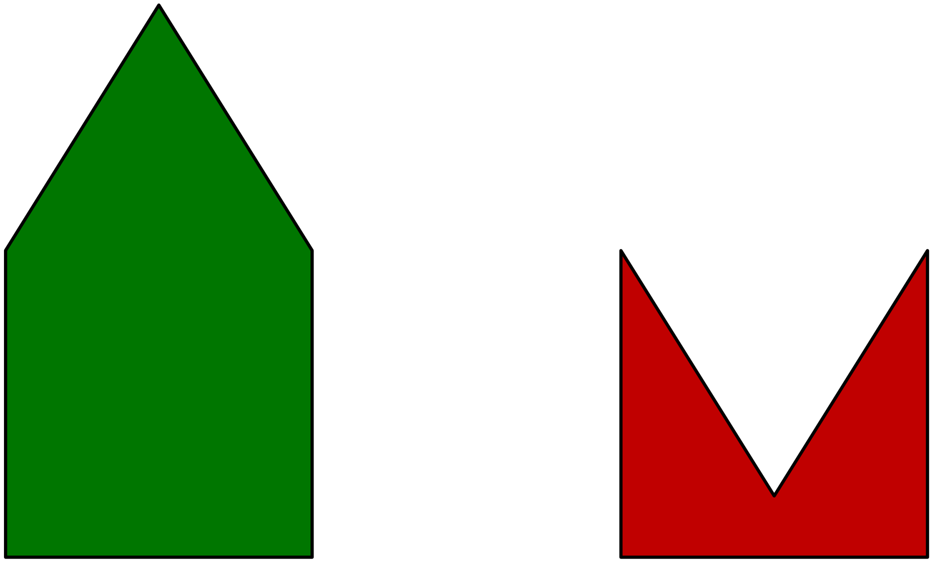
We are driven by our ideas ... .

1 What would we be without imaginations?

1 Imagine this

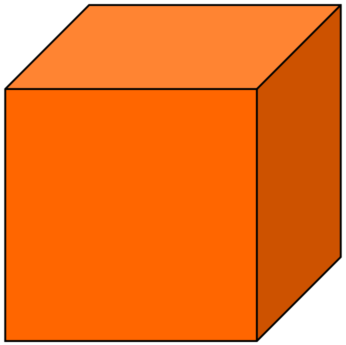
A pentagon with two adjacent right angles and five equidistant sides is called a **beautiful pentagon.**

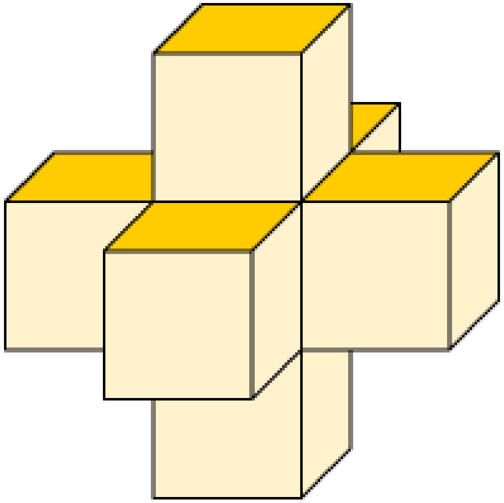
Draw a beautiful pentagon!



1 Imagine this

Imagine a body whose faces are all squares.





A cube is a body whose faces are all squares.

Is that a definition of the concept "cube"?

1 We imagine …

- Objects and their properties.

  E.g., a cube whose opposite faces are painted in the same color.

- Relations between objects

  E.g., a cube on a cylinder.

- processes

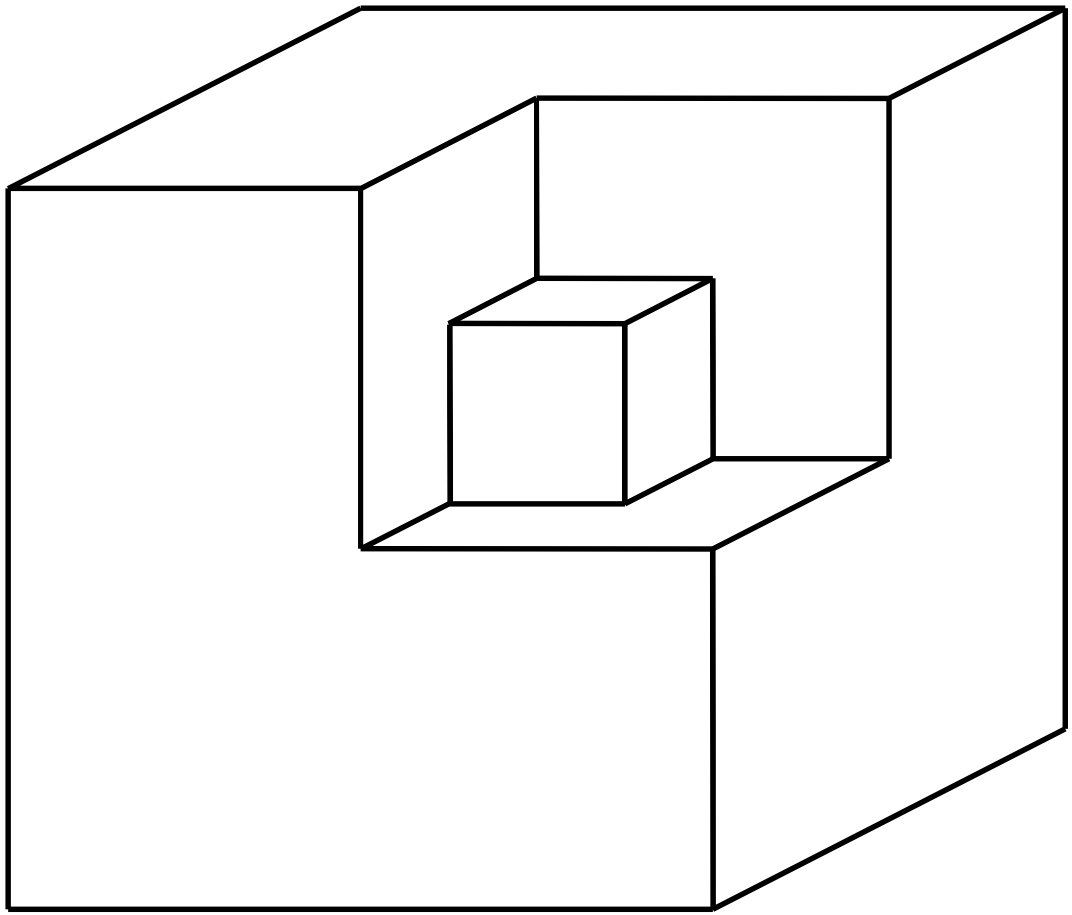
  E.g., Paul builds a tower of cubes.

1 Importance of spatial abilities

(1) We are living in a 3D-world

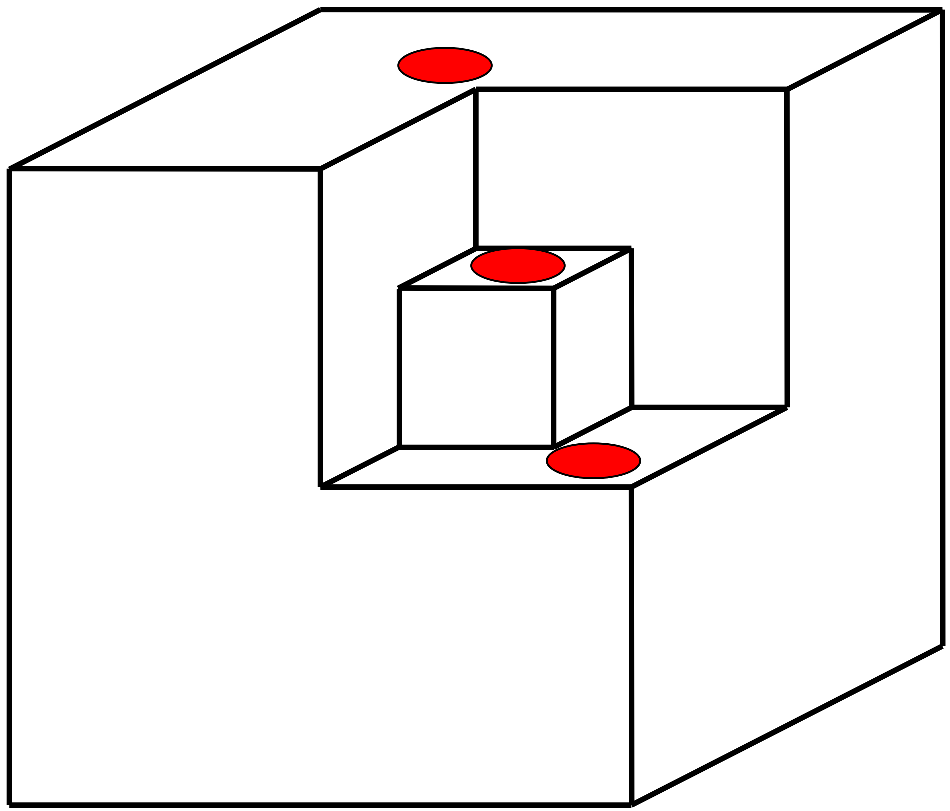
- Our world is a 3D-world
- All our possibilities to describe the 3D-reality are verbal or 2D.
- That's why we always need the ability to visualize 3D objects using verbal descriptions or 2D images.
- What do you see in the following picture?

1 What do you see here?



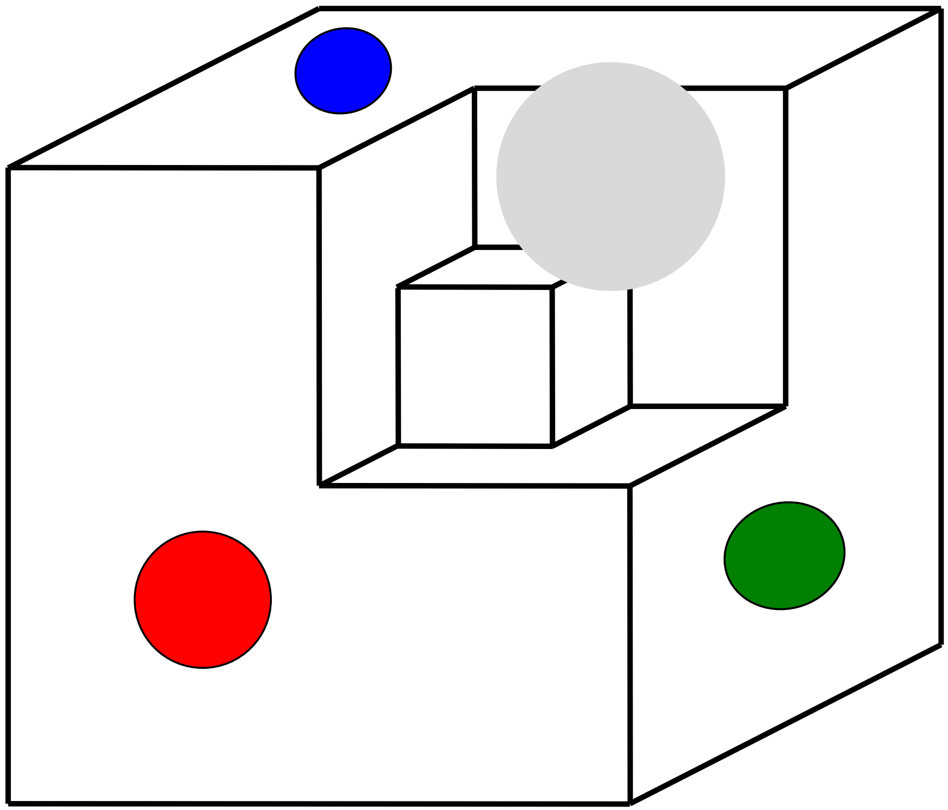
1

What do you see here?



A cube was cut out of a cube. There's a little cube in there.
The red circles mark a “stairway”.

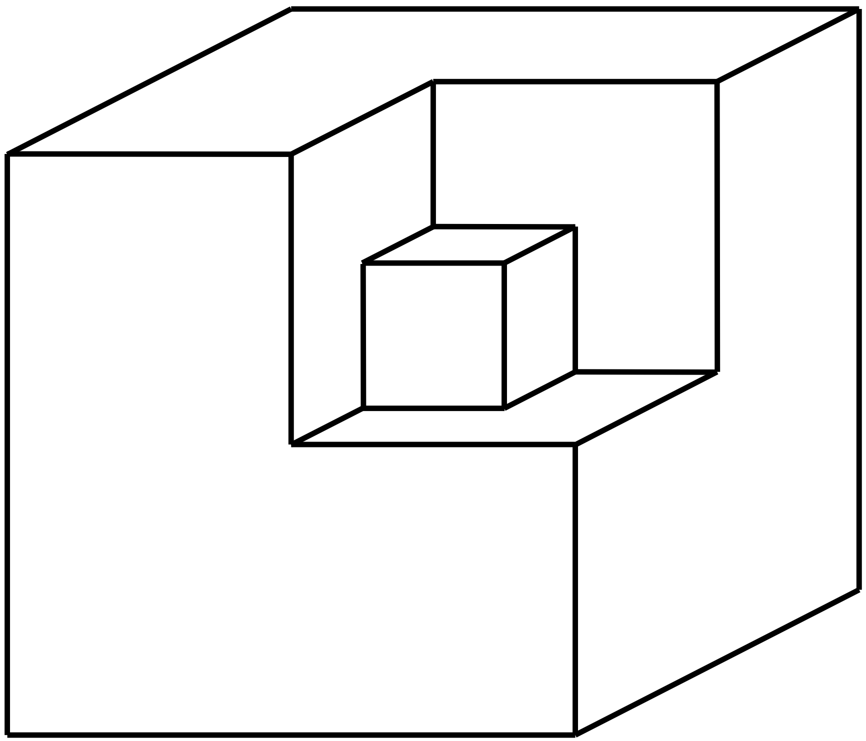
1 What do you see here?



Do you see a room? There's the back wall.
There’s the right wall. There's the ceiling.
The right upper back corner is not visible.
There hangs a cube.

1

What do you see here?

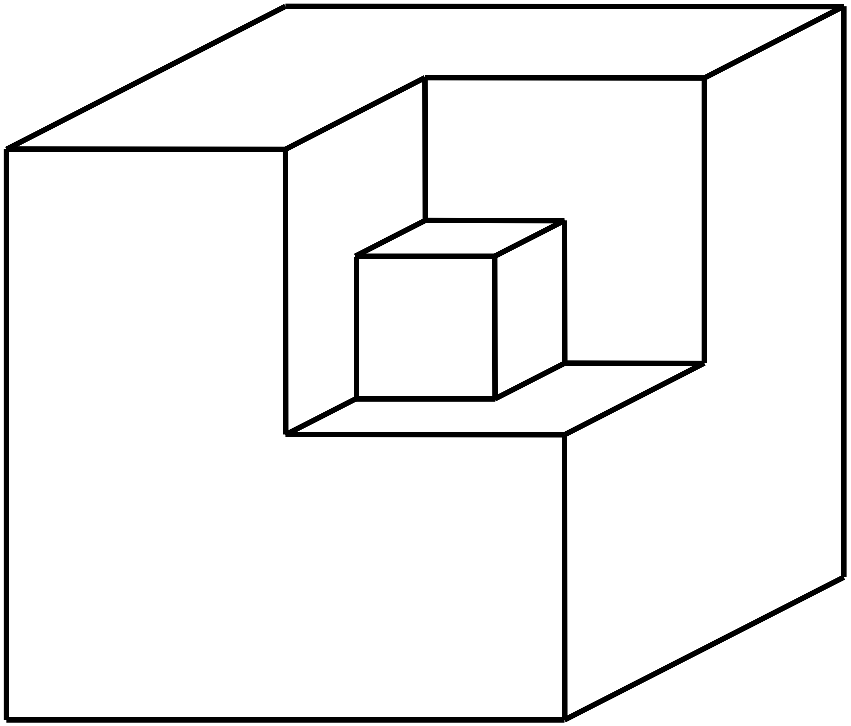


What did you really see? A stairway? A room?

NO! You see black lines on a white wall, a square etc.

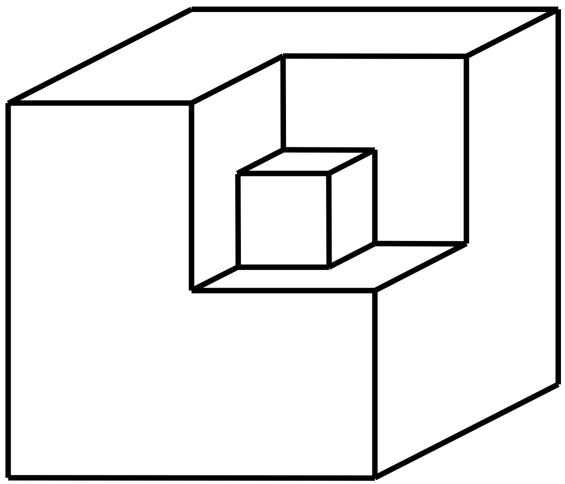
The stairway and the room were imaginations!

1 What do you see here?



Which imagination is easier? The imagination of a stairway or the imagination of a room?

1 What do you see here?

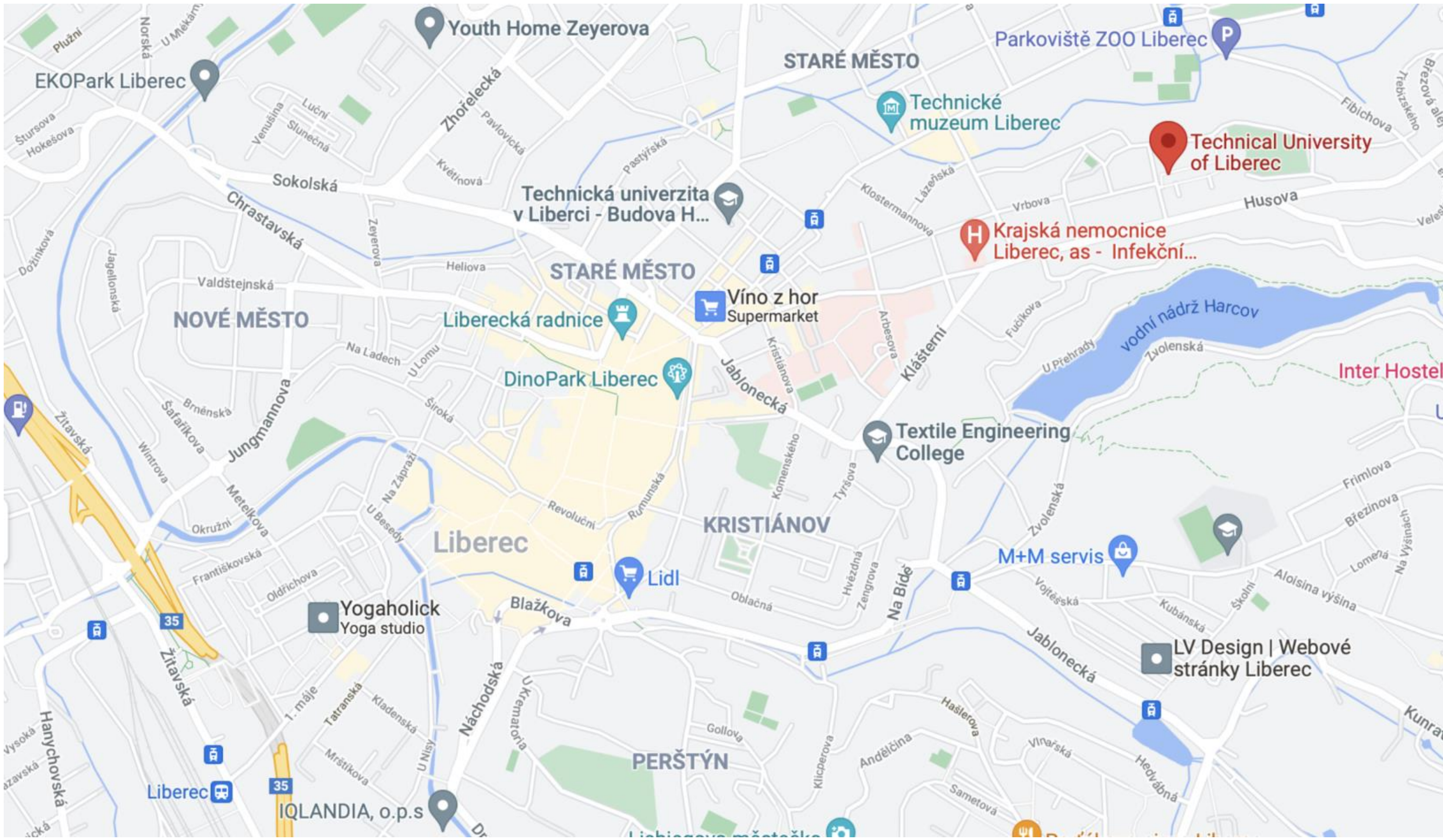


The explanation: If a 2-D perception can evoke different 3-D imaginations, then the 3-D imagination becomes dominant, which corresponds to the more common 3-D perception.

We perceive stairways etc. much more often than things in the upper right corner.

1 Importance of spatial abilities

(2) Daily life: using maps



Describe the way from the University to the train station.

1

Importance of spatial abilities

(3) Success in profession

- There is a **correlation between success in profession and spatial-visual abilities**.
- This is not only true for architects, designers etc.
- The reason is, that persons with good spatial-visual abilities organize their work better!

1 Importance of spatial abilities

(4) Arithmetic perfomance

There is a relation between dyscalculia and a lack of spatial abilities.

(e.g. J.-H. LORENZ 1992)
(see also Dehaene – SNARC - Effect ...)

2 Spatial ability

- Concept “spatial ability” (“spatial-visual qualification”)

- Five components of spatial ability
  - spatial perception
  - Imagination of spatial relations
  - spatial orientation
  - Visualization
  - Imagination of rotations

- Knowledge about the development of spatial ability

2 Components of spatial ability

2.1 Spatial perception

is the ability to ***perceive*** spatial objects and relationships in 3D reality

- perceive the relationships between the location of objects and one’s own person.
- grasp and split the 3D reality using horizontal planes, vertical planes and planes parallel to the plane of the viewer's face

2 Components of spatial ability

2.2 Spatial relations

The ability to ***imagine*** the spatial relationships between objects (e. g. behind and in front, on top and below, to the right of, to the left of, between...)

The person is ***outside*** the situation.

2 Components of spatial ability

2.3 Spatial orientation

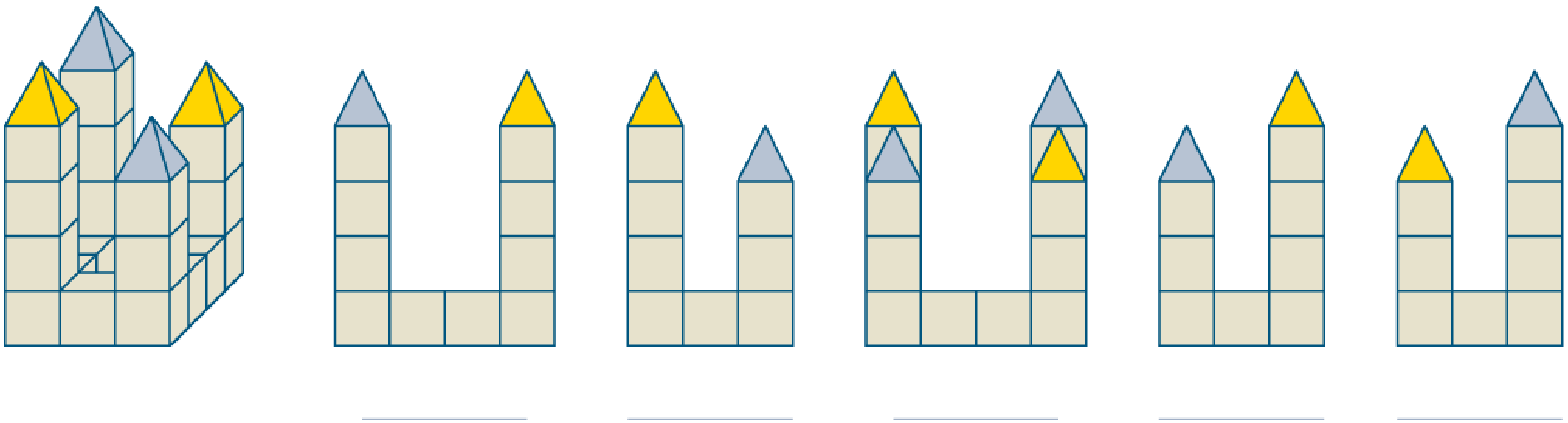
The ability to mentally arrange oneself into a spatial situation and to imagine what is right, what is left etc.

It is the ability to orientate oneself with a map or a site plan.

It is also the ability to move mentally in this situation, this map.

2 Components of spatial ability

2.3 Spatial orientation



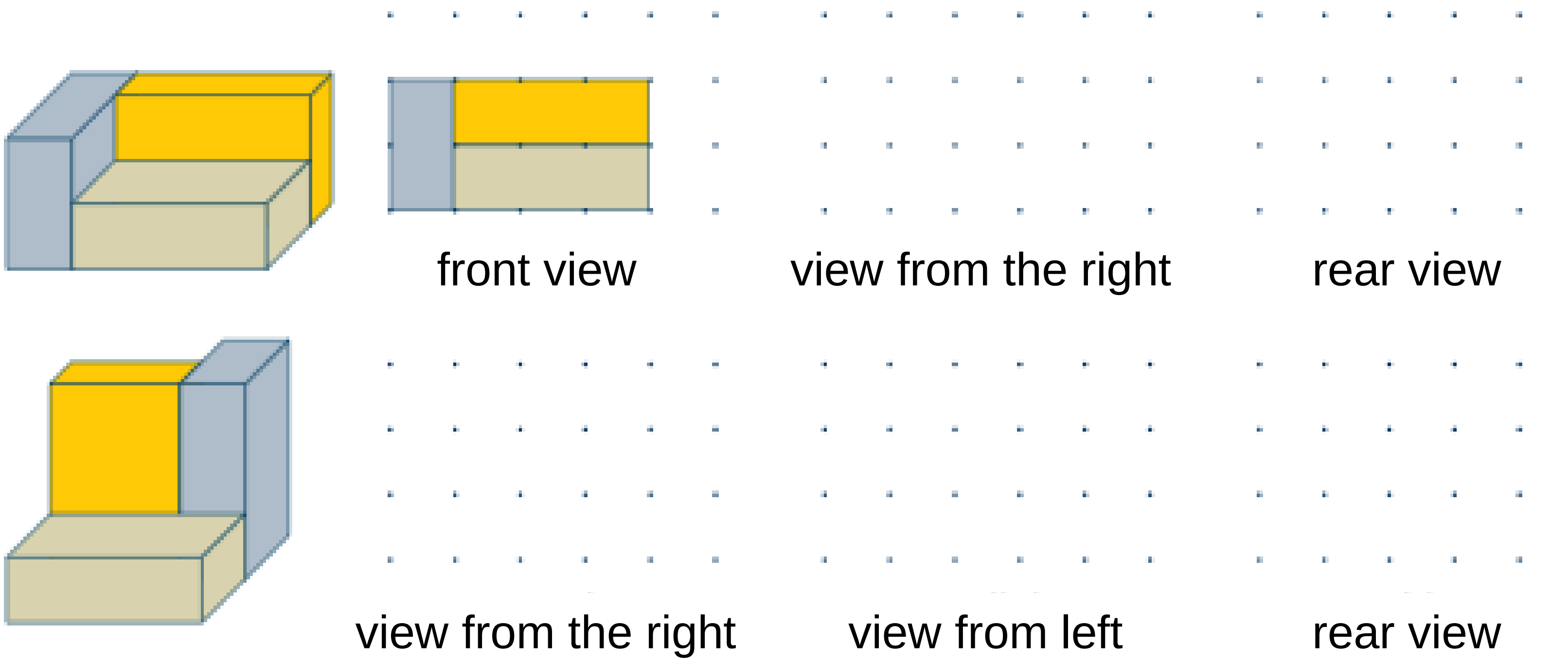
Which images are views of the castle?

From which side are you seeing it?

(In this task it’s only necessary to evaluate a given view.)

2 Components of spatial ability

2.3 Spatial orientation



It’s a difficult task because the view must be constructed mentally.
That’s much more than the evaluation of a given view.

Components of spatial ability

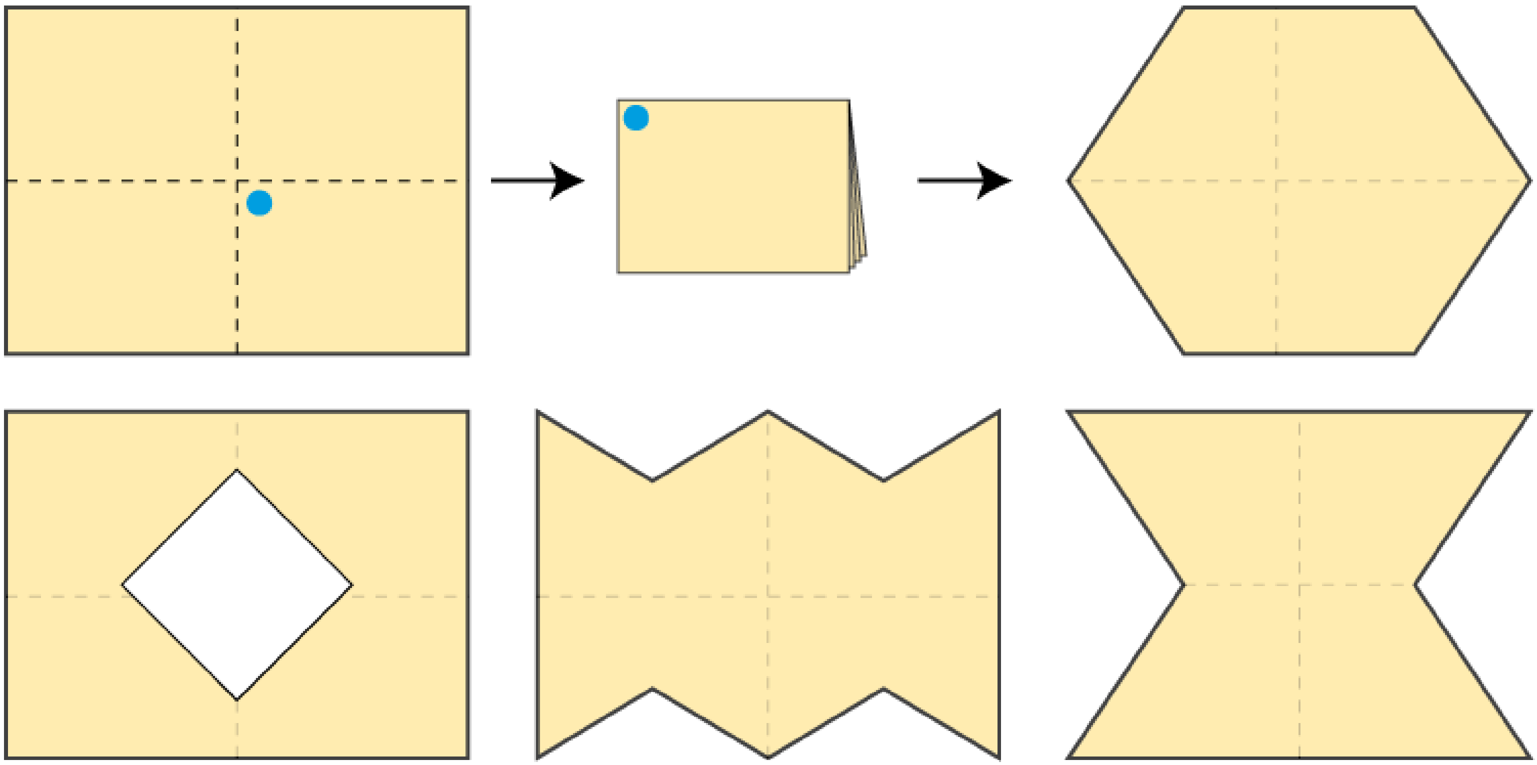
2.4 Spatial Visualization

is the ability to operate mentally with objects, fold them, disassemble them, color them, move them, etc. –
only not to rotate.

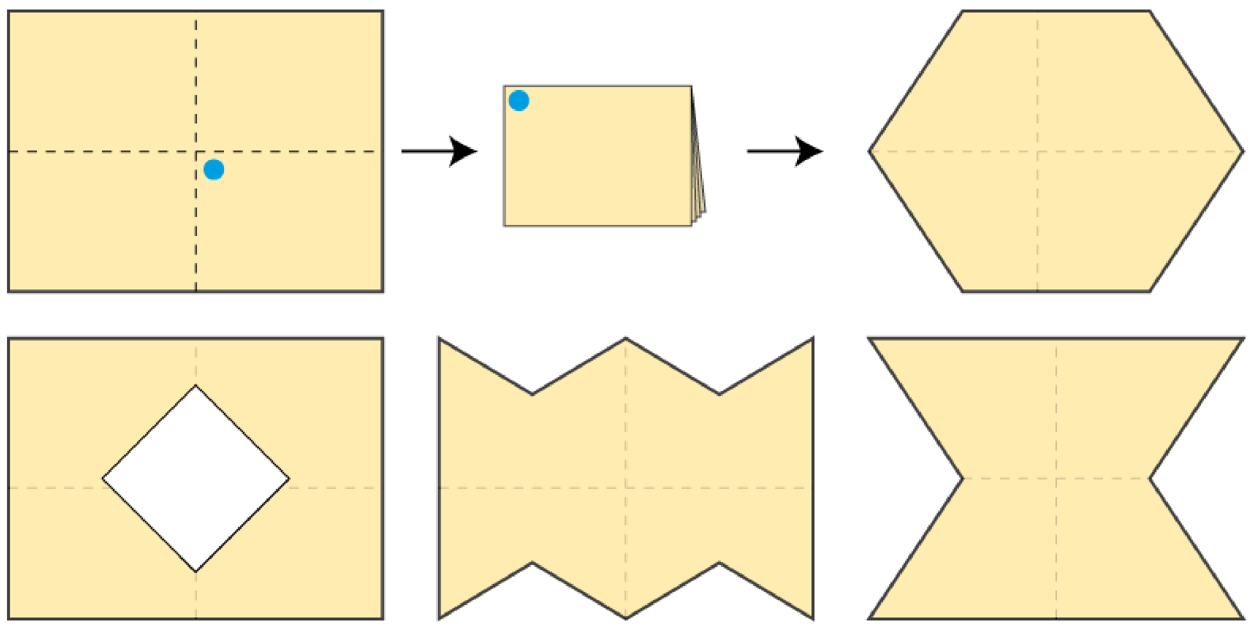
2 Components of spatial ability

2.4 Spatial Visualization

How do you must cut the paper to make these figures?



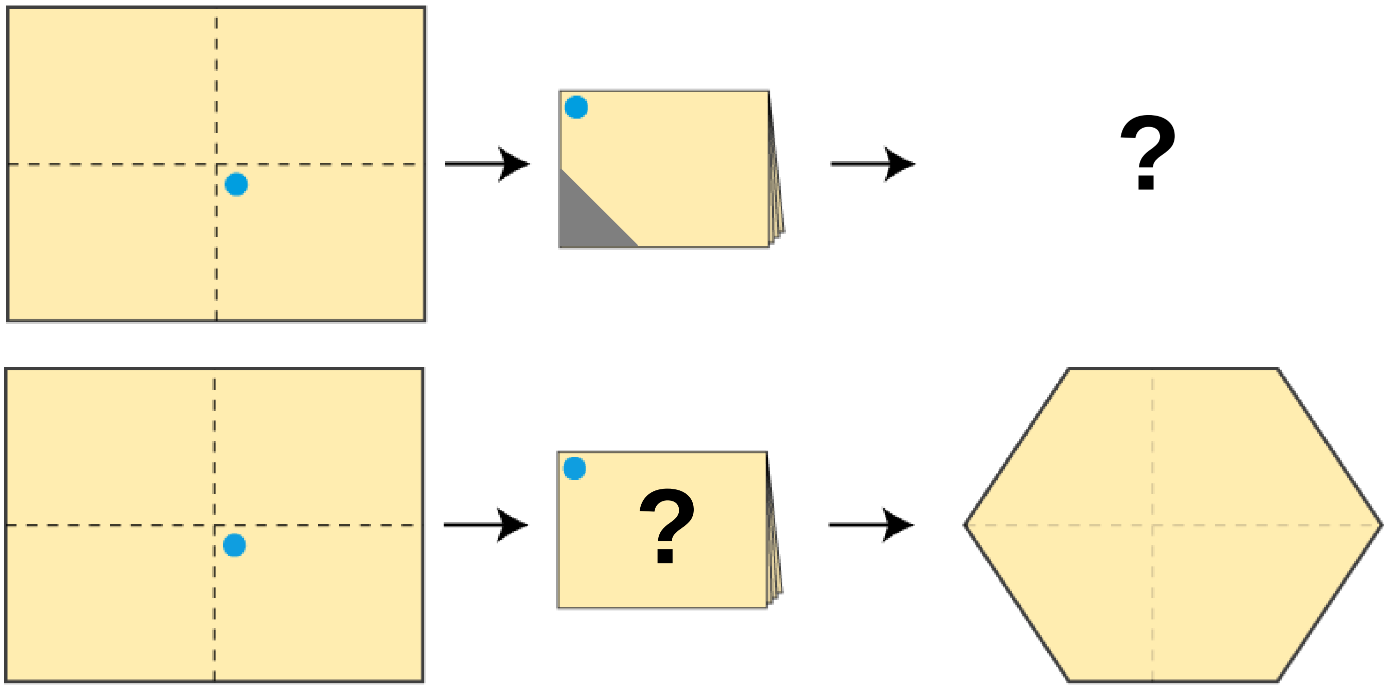
2.4 Spatial Visualization



When the sheet of paper is folded twice, the blue dot marks the “closed” corner.
This is important so that all students lay the paper in the same orientation and cut at the same corner (side).

2 Components of spatial ability

Didactical core: There are two types of tasks

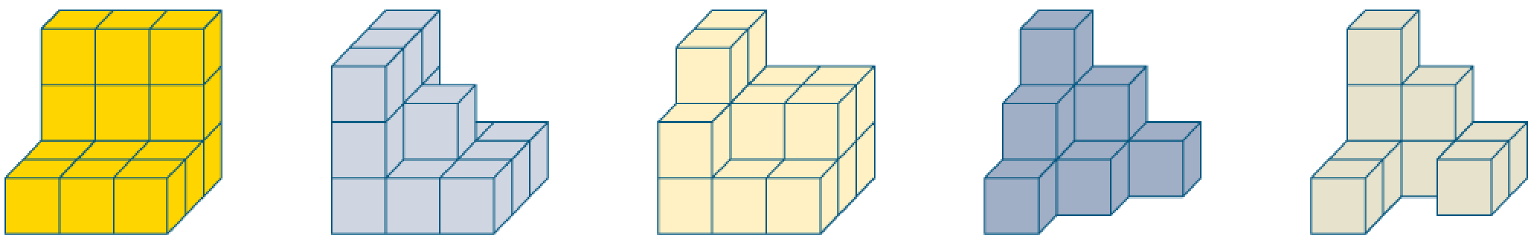


Starting state (folded paper)	Process (cutting)	final state (Paper with a cut)
given	given	?
given	?	given

2 Components of spatial ability

2.4 Spatial Visualization

It was a 3 x 3 x 3 cube. How many cubes have been taken away?

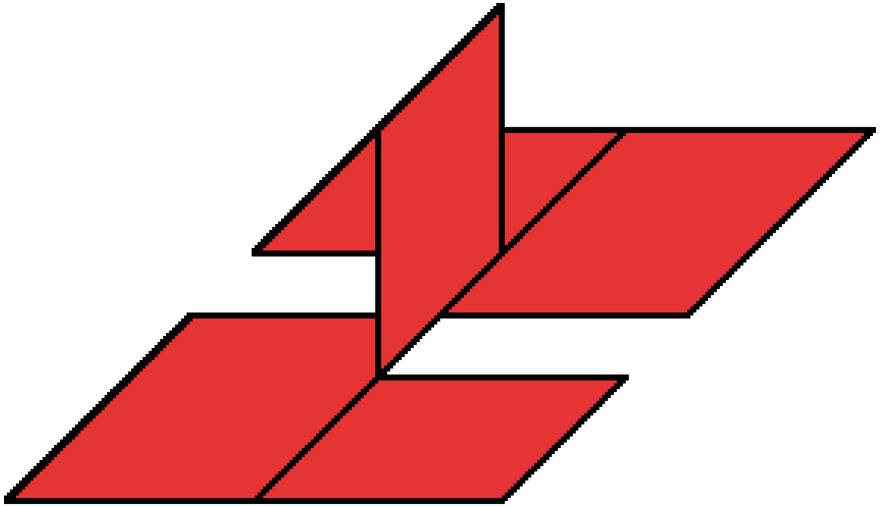


What ways do students use to solve the problem?
Students should describe their ways.
What ways to solve the problem they can use?

Components of spatial ability

2.4 Spatial Visualization

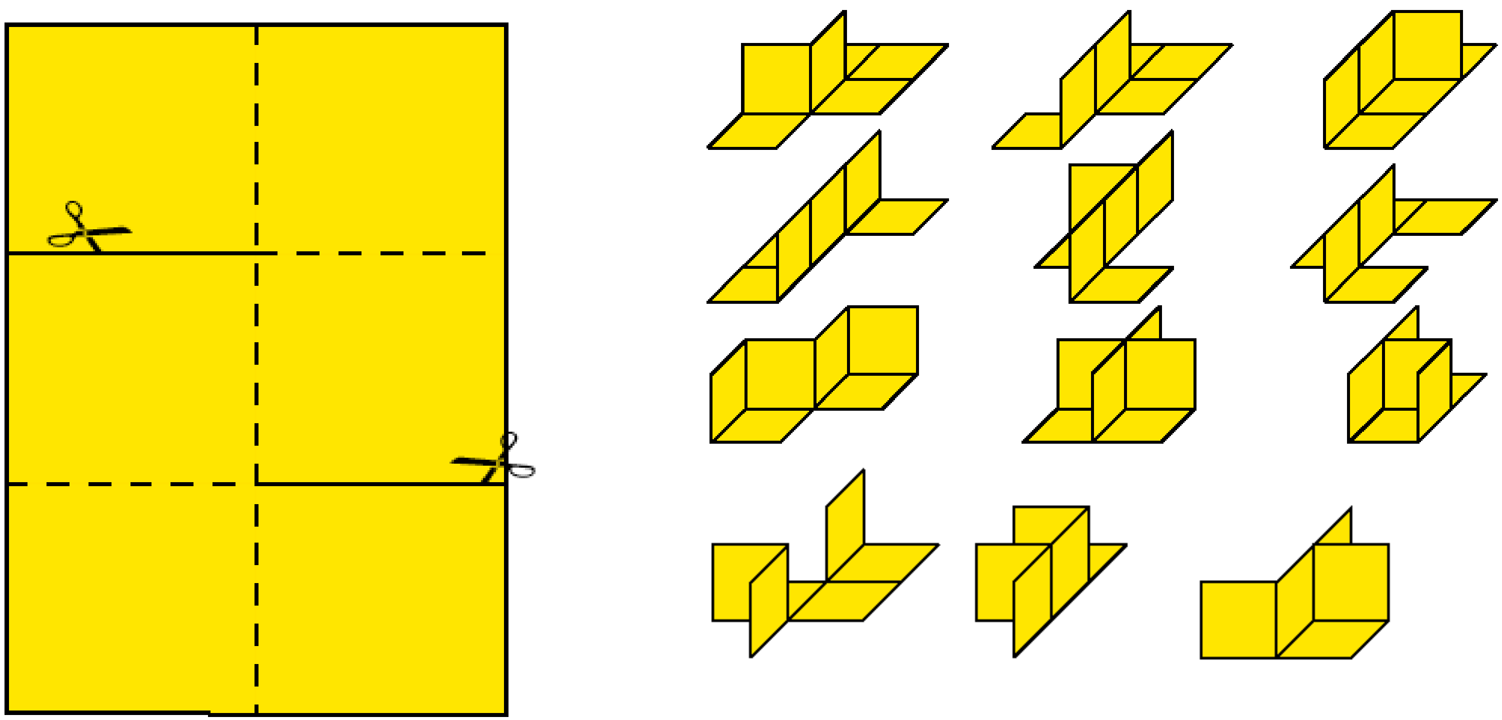
Try that. How do you have to cut and fold?



Components of spatial ability

2.4 Spatial Visualization

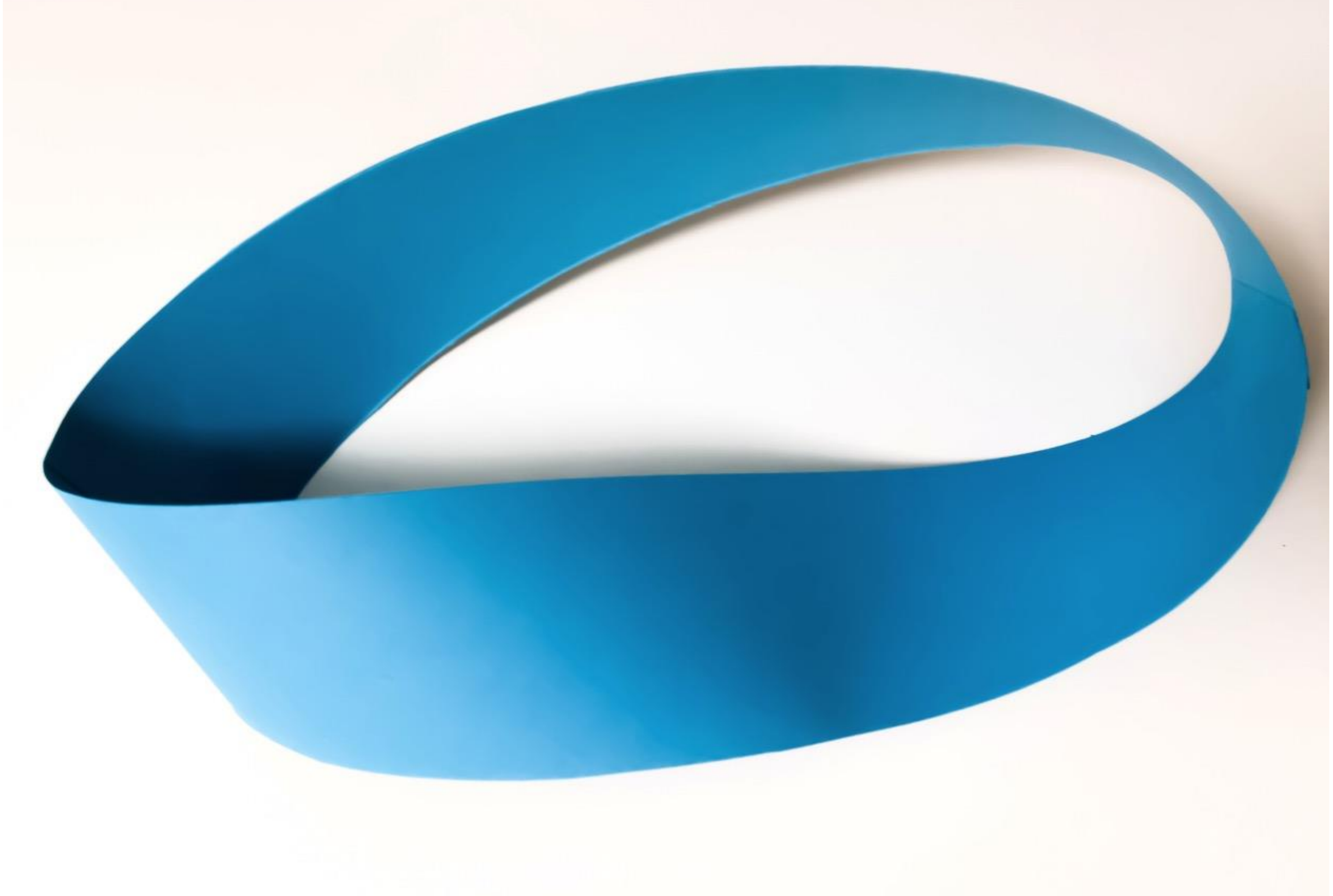
Fold along the dashed lines. Which figures you can’t fold? Imagine it first and decide, then try and proof.



2 Components of spatial ability

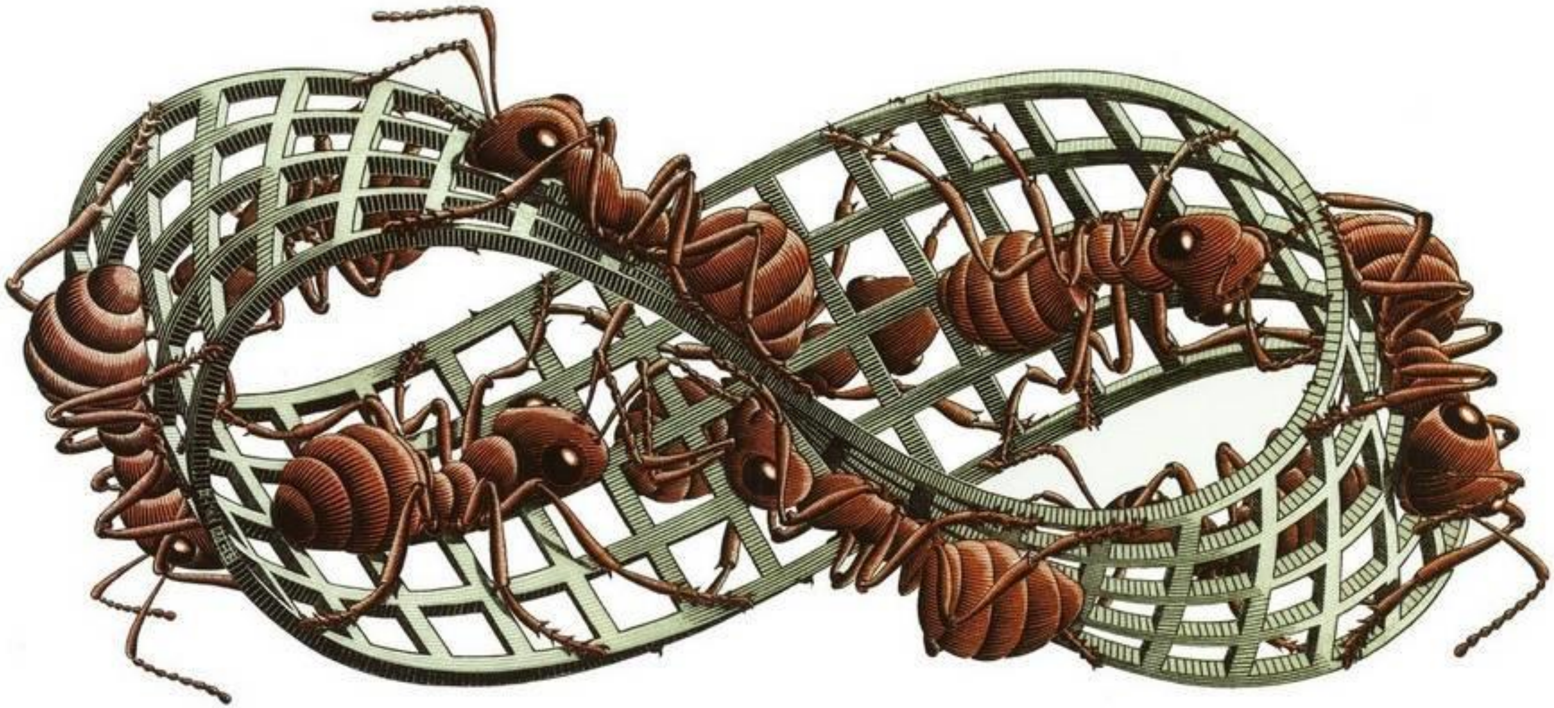
2.4 Spatial Visualization

Imagine how an ant runs along the Möbius strip.



2.4 Spatial Visualization

All ants are on the same side of the strip!



M.C. Escher: “Möbiusband II (rote Ameisen)", ca. 1963

2 Components of spatial ability

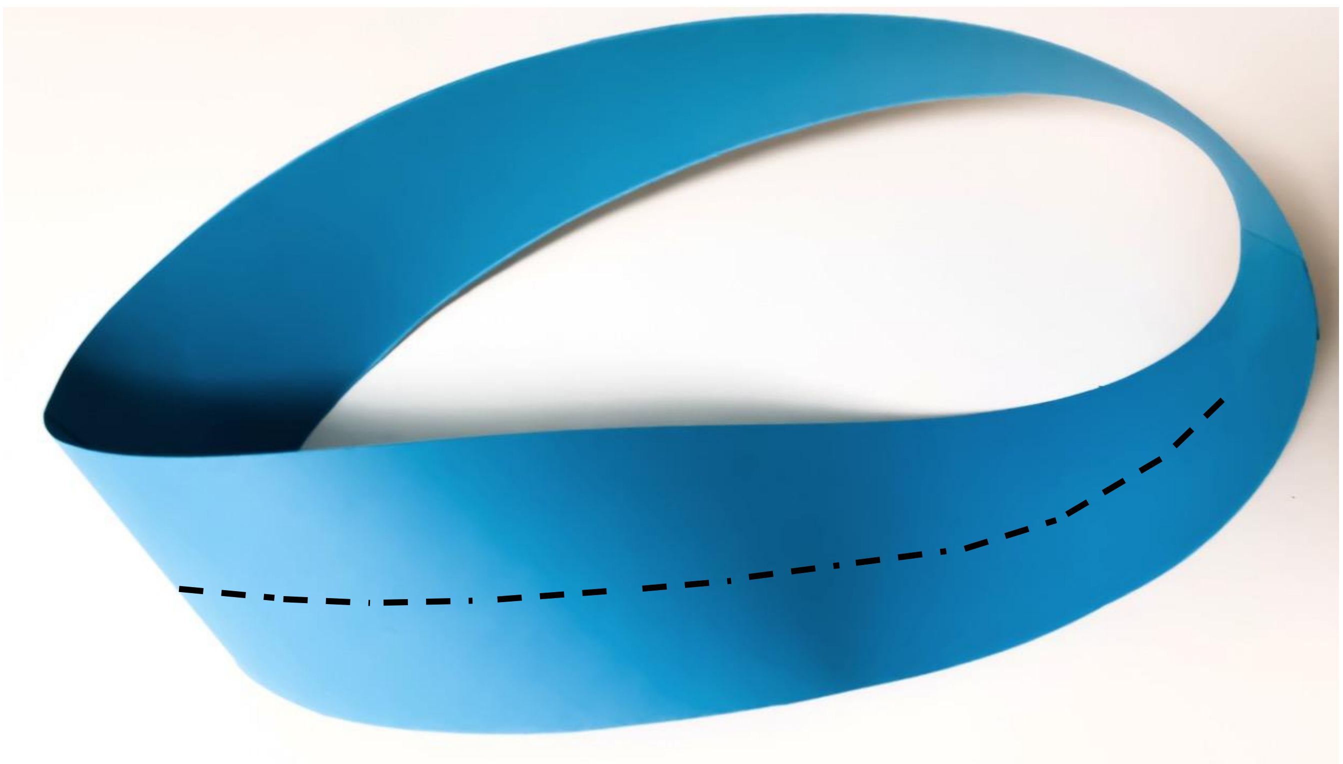
2.4 Spatial Visualization

If an ant runs along this strip, it will return to the starting point …

2 Components of spatial ability

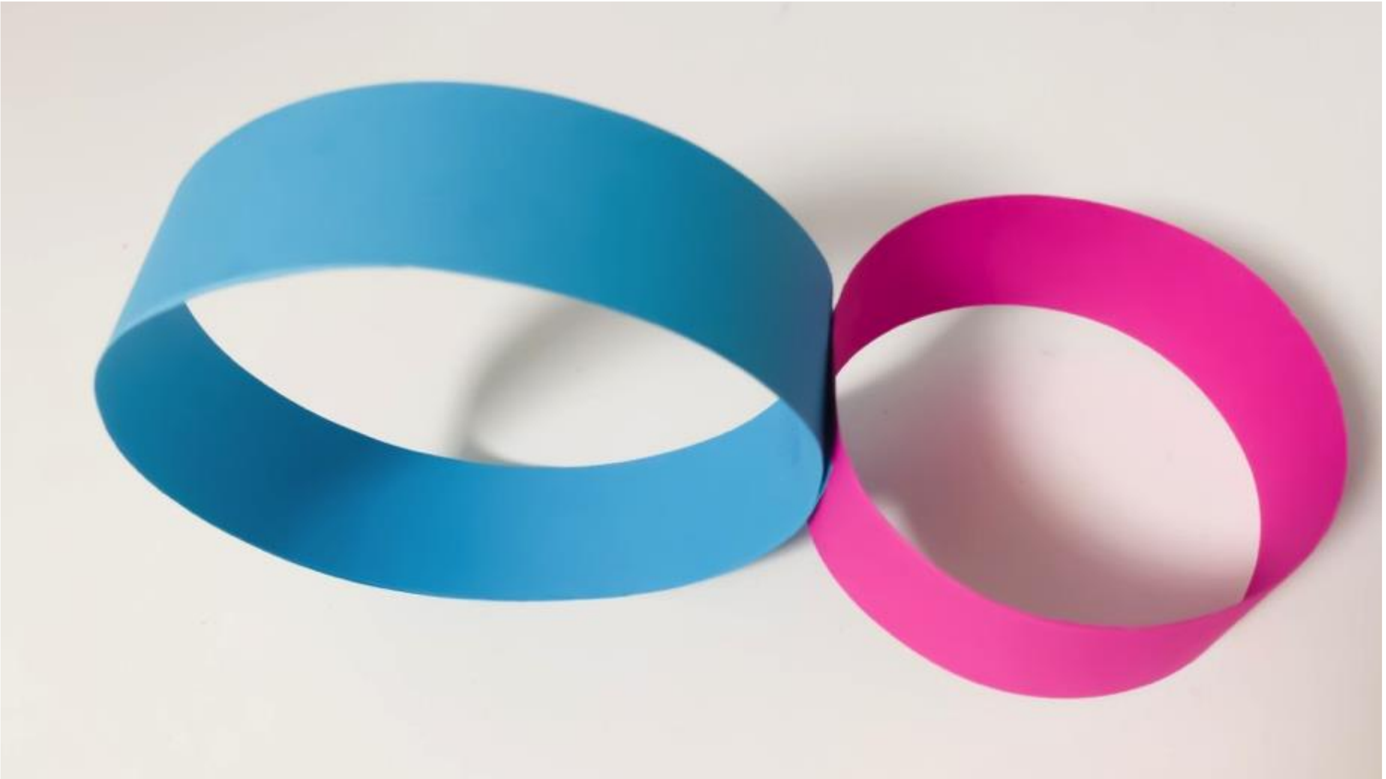
2.4 Spatial Visualization

Imagine the result when you cut the Möbius-strip lengthwise in half!
First imagine it, describe and explain it, then try it out.



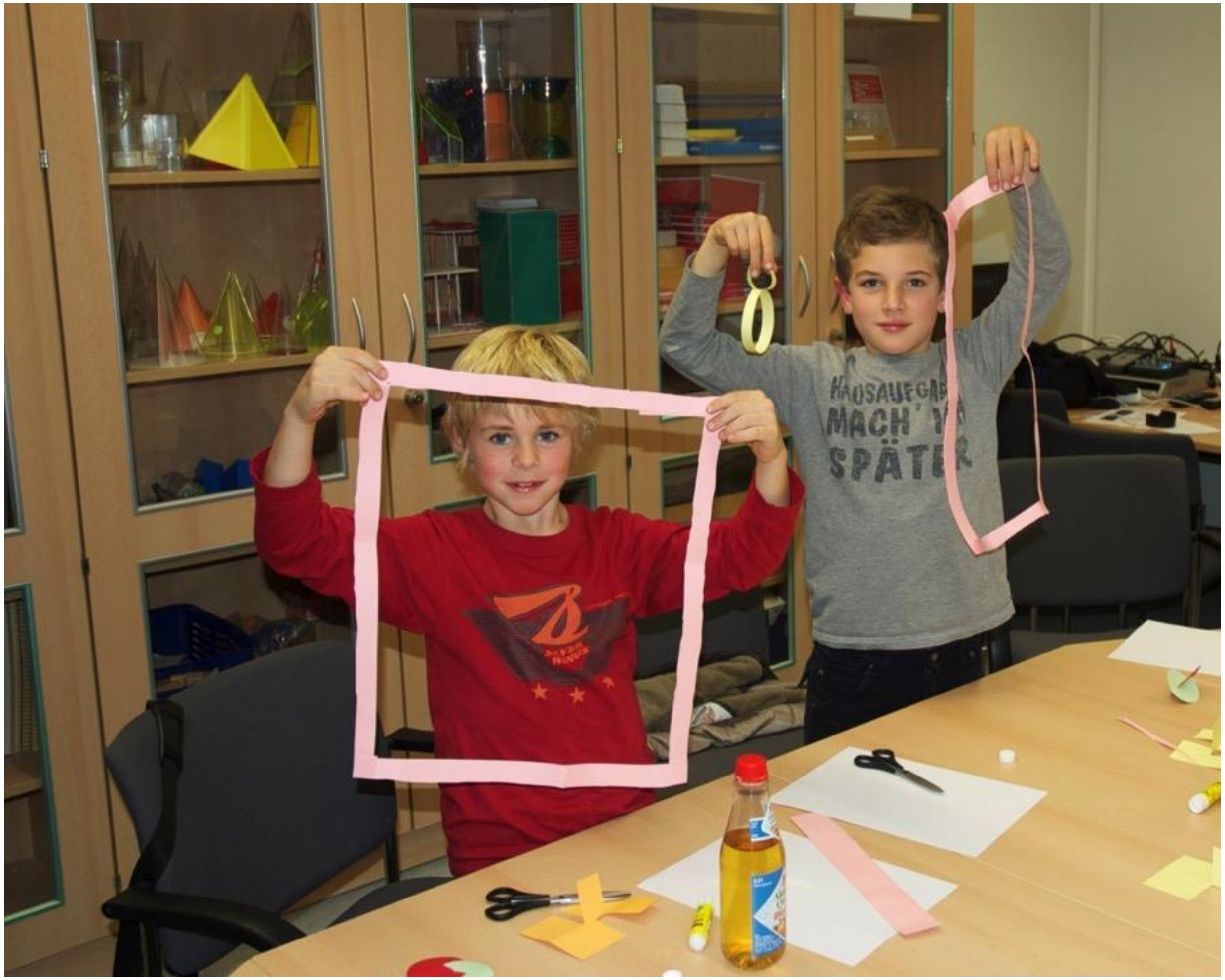
2.4 Spatial Visualization

Glue the two rings together.
Then cut so, that you get a rectangle.



2 Components of spatial ability

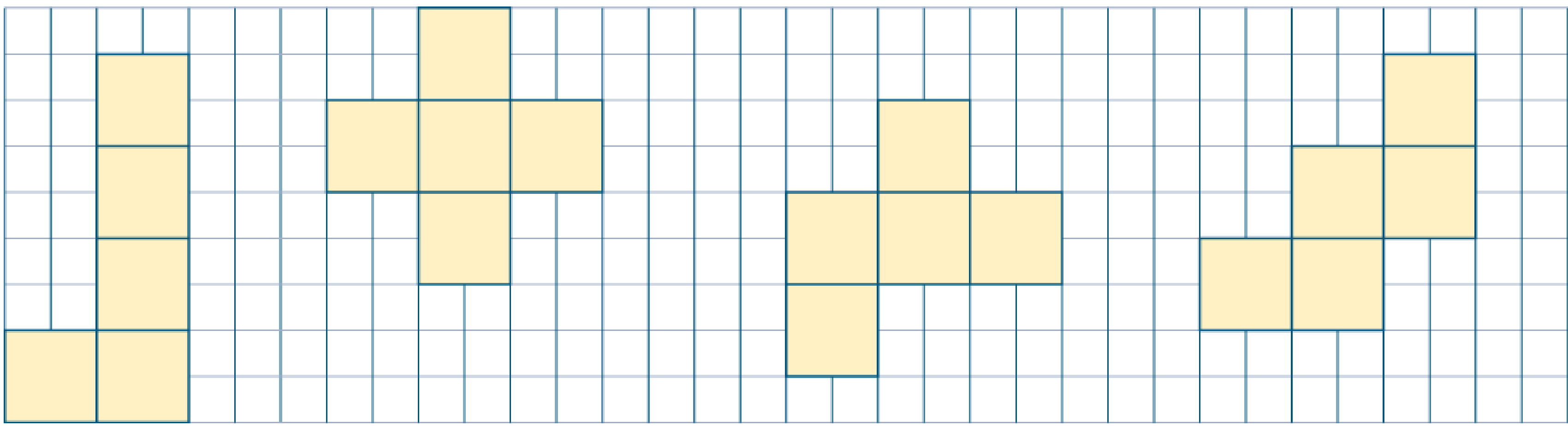
2.4 Spatial Visualization



Components of spatial ability

2.4 Spatial Visualization

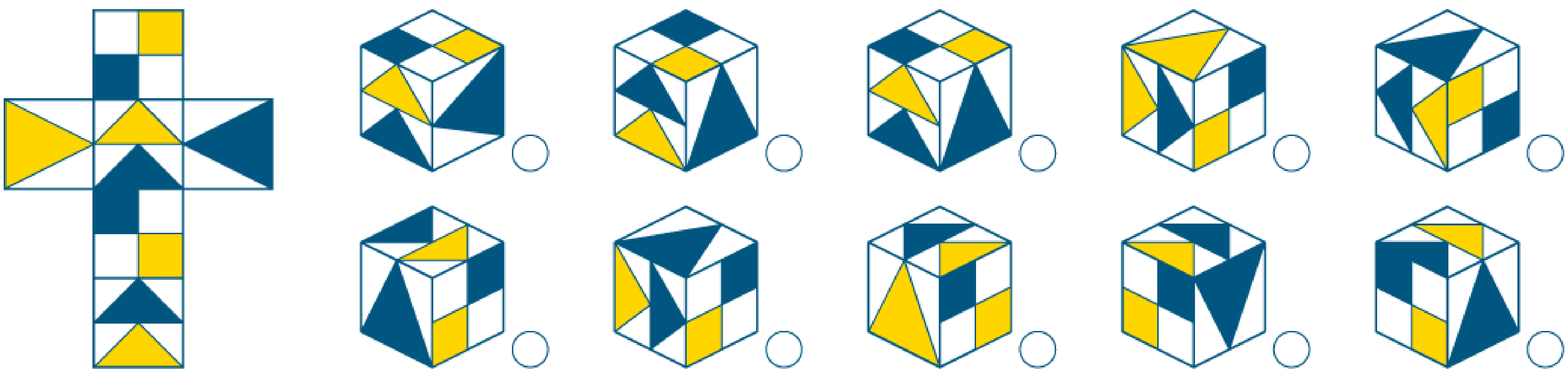
Complete the figures to nets of cubes.



2.4 Spatial Visualization

Which cubes fit the net? Mark and explain first.

Then check by cutting out and folding the net.



2.5 Imagination of rotations

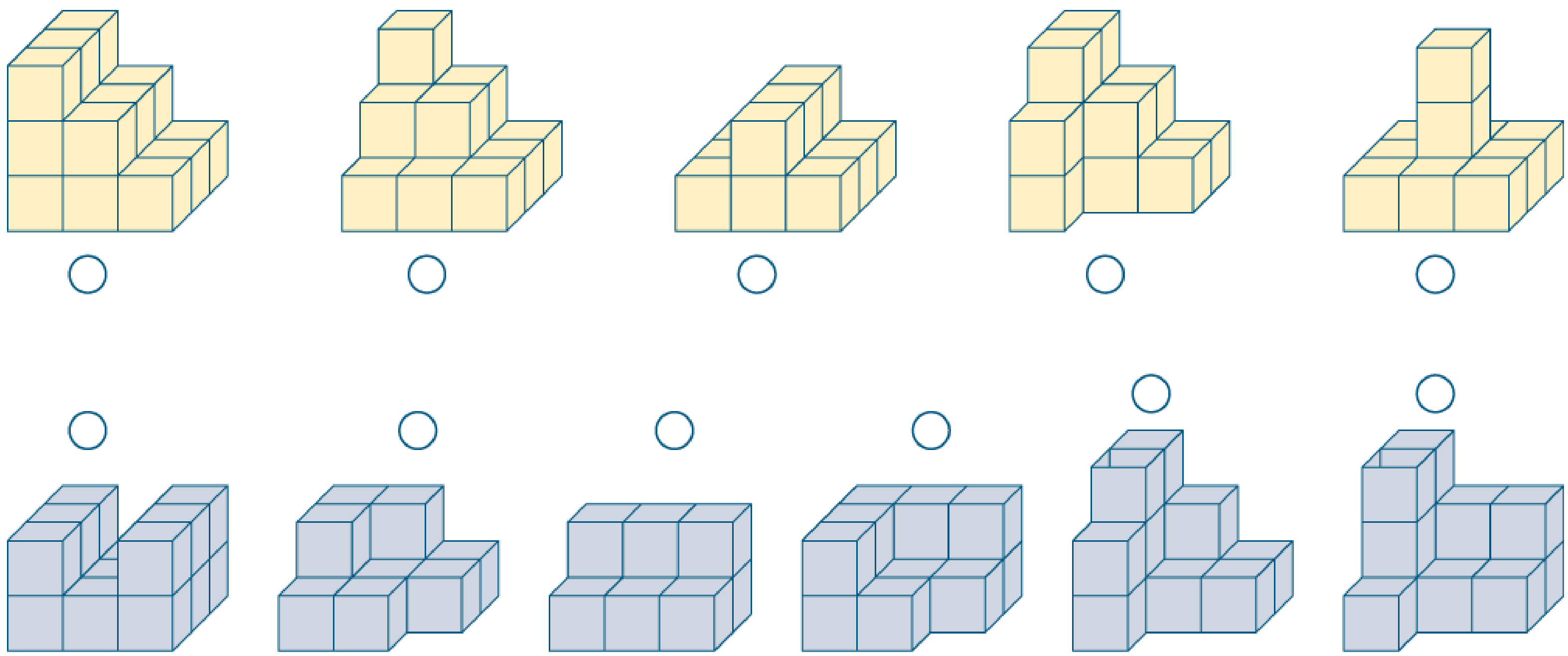
is the ability to imagine quickly and precisely a rotation of an object around an axis.

(the axis can be horizontal, vertical, an axis perpendicular to the horizontal and the vertical axis or a combination).

2 Components of spatial ability

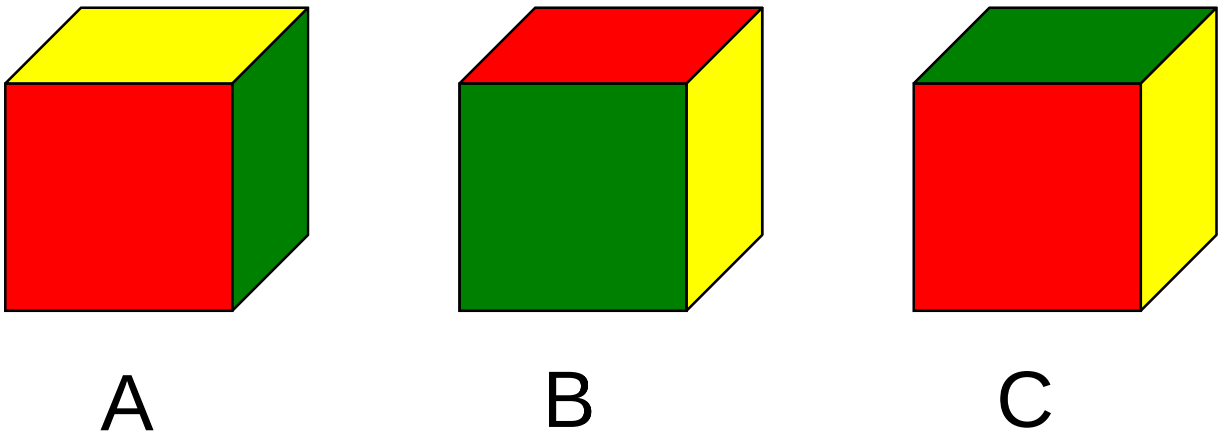
2.5 Imagination of rotations

A yellow and a grey part should be built together to make a cube. Find matching parts.



Components of spatial ability

2.5 Imagination of rotations



All no visible faces of the cubes are black.

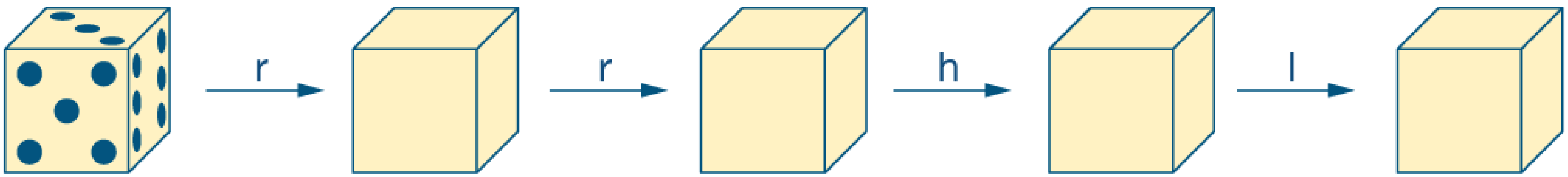
How you must rotate the cube

- from position A to position B
- from position A to position C

2 Components of spatial ability

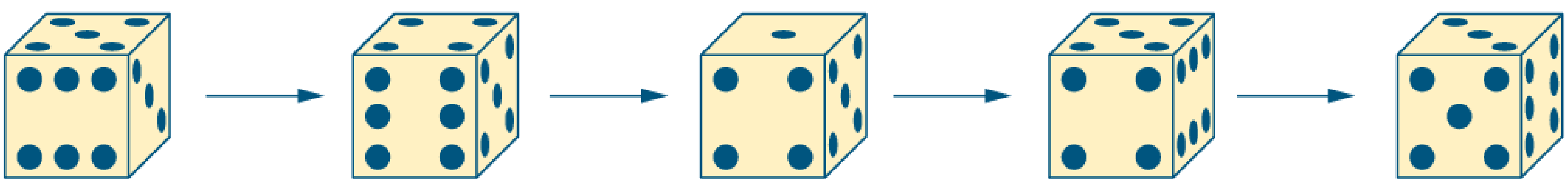
2.5 Imagination of rotations

The dice is tilted. Draw in the dots correctly.



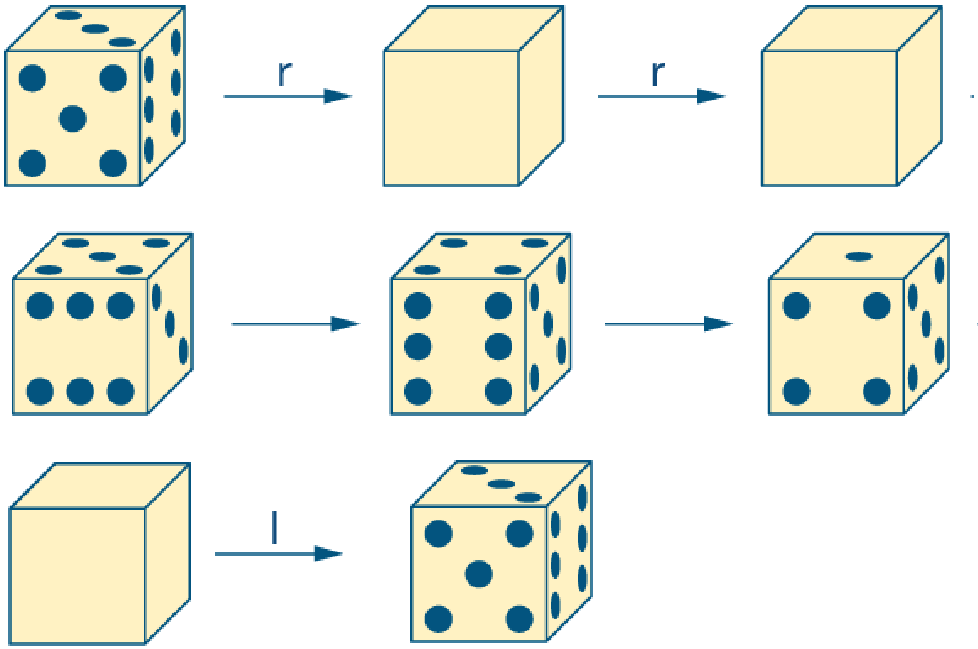
(r – tilt to the right, b – tilt back, l – tilt to the left, f – tilt forward)

How do you have to tilt the dice? (Write r, b, l, or f)



Components of spatial ability: Rotation

Didactical core: There are three types of tasks:



Starting state	Process	final state
given	given	?
given	?	given
?	given	given

3 Software to develop spatial ability

3.1 problems working in the traditional way

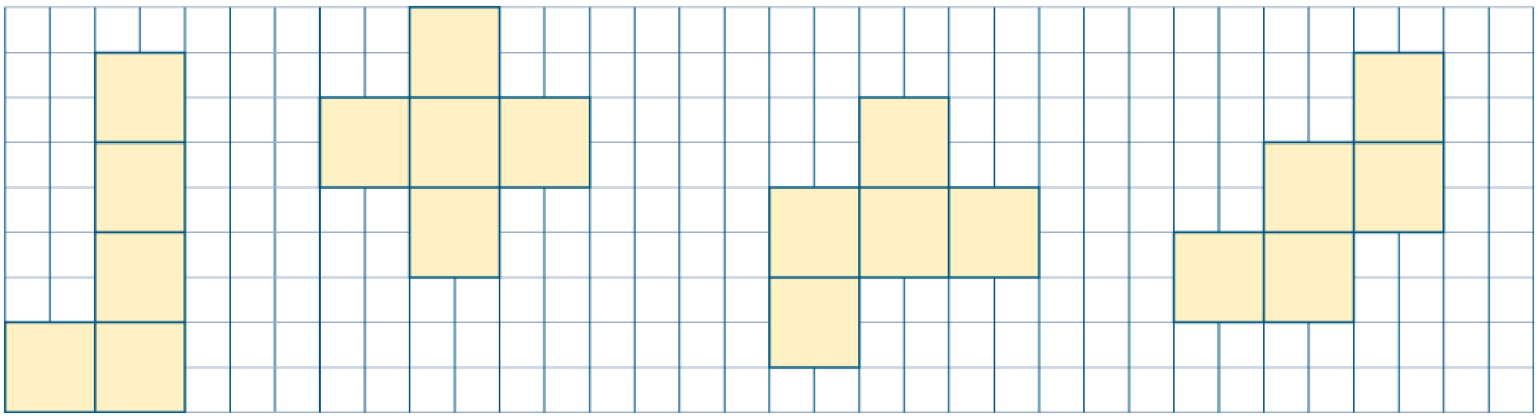
A lot of time for some other activities beyond activities that promote spatial ability.

Students spent time for drawing, cutting, folding **...**

3 Software to develop spatial ability

3.1 problems working in the traditional way

Complete the figures to nets of cubes.



The goal is to develop spatial visualization.

The students imagine the solution, then draw, cut, and fold to check.

Thus, they spend a lot of time for drawing, cutting, folding …

3

Software to develop spatial ability

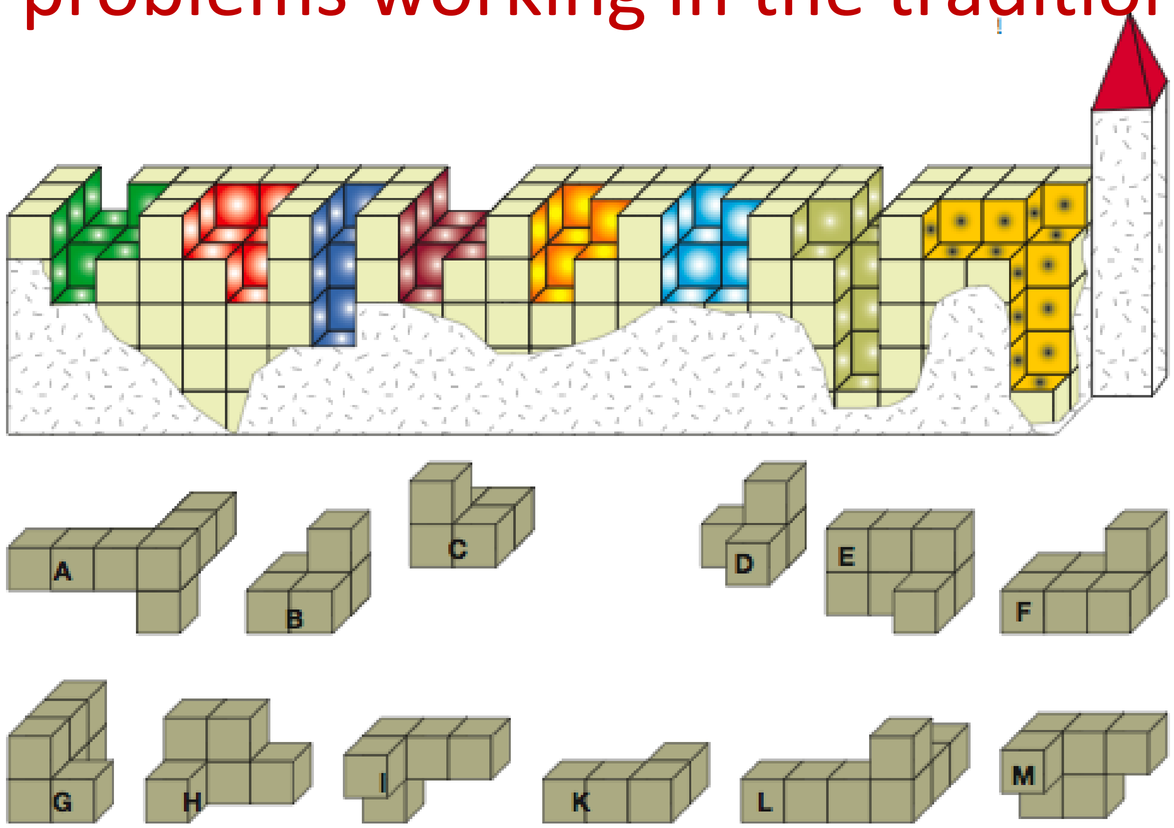
3.1 problems working in the traditional way

Tasks for developing spatial imagination need ***often much material*** with which children can check their imagination.

Such material is usually not available with adequate quality and quantity.

3 Software to develop spatial ability

3.1 problems working in the traditional way



Which part fits into which position? (From a textbook)
The student has no possibility to proof and see!

3 Software to develop spatial ability

3.1 problems working in the traditional way

Software to develop spatial ability

3.2 possibilities

- Fast graphics cards with interfaces such as Open GL allow real-time rendering of spatial processes.
- Fast Internet allows animations without long loading times in the browser.
- Touchscreens replace mouse and keyboard.
- Student can control the software by touching the objects directly with the finger. It's not necessary to learn how to operate keyboard and mouse in context of the software.

3

Software to develop spatial ability

3.2 possibilities

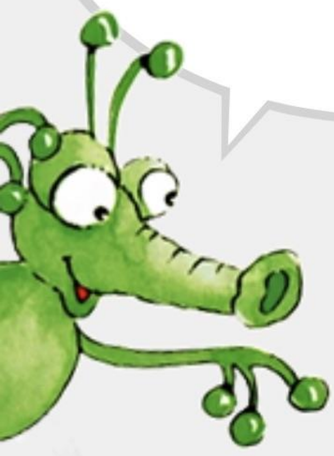
We hope, software can help to use working time effectively.

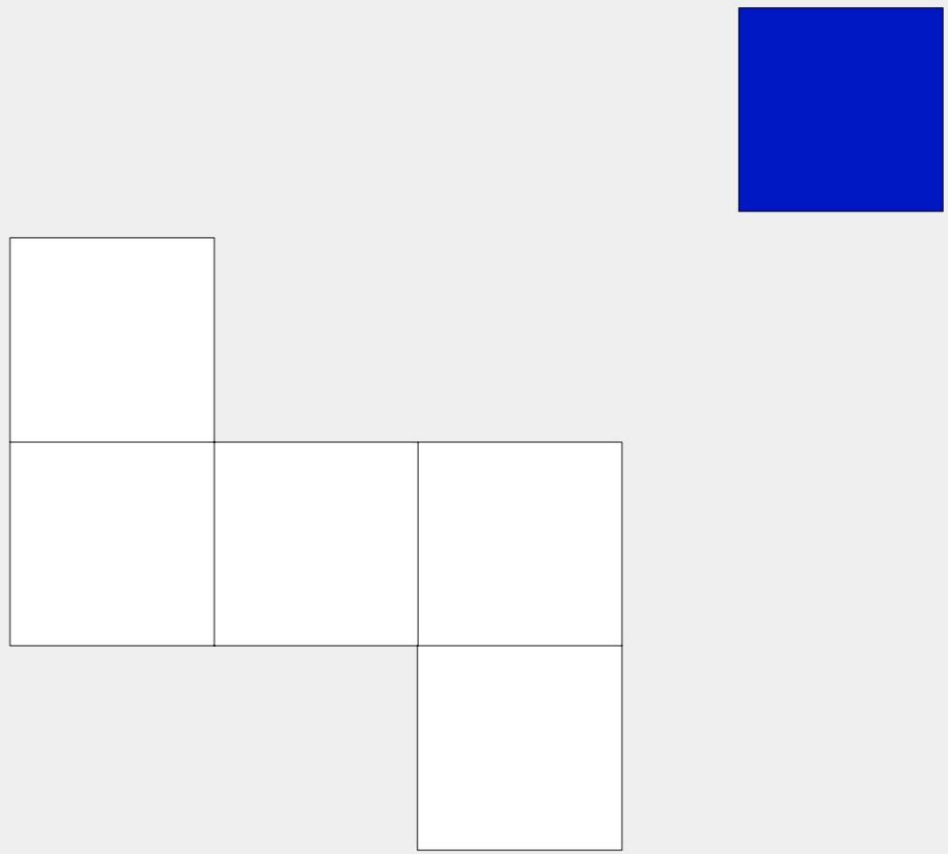
Example: Completing to cube nets

- The student trains his imagination
- The software provides informative feedback.
- Time-consuming activities such as drawing, cutting, and folding are no longer necessary.
- The student can spend all working time on visualization.

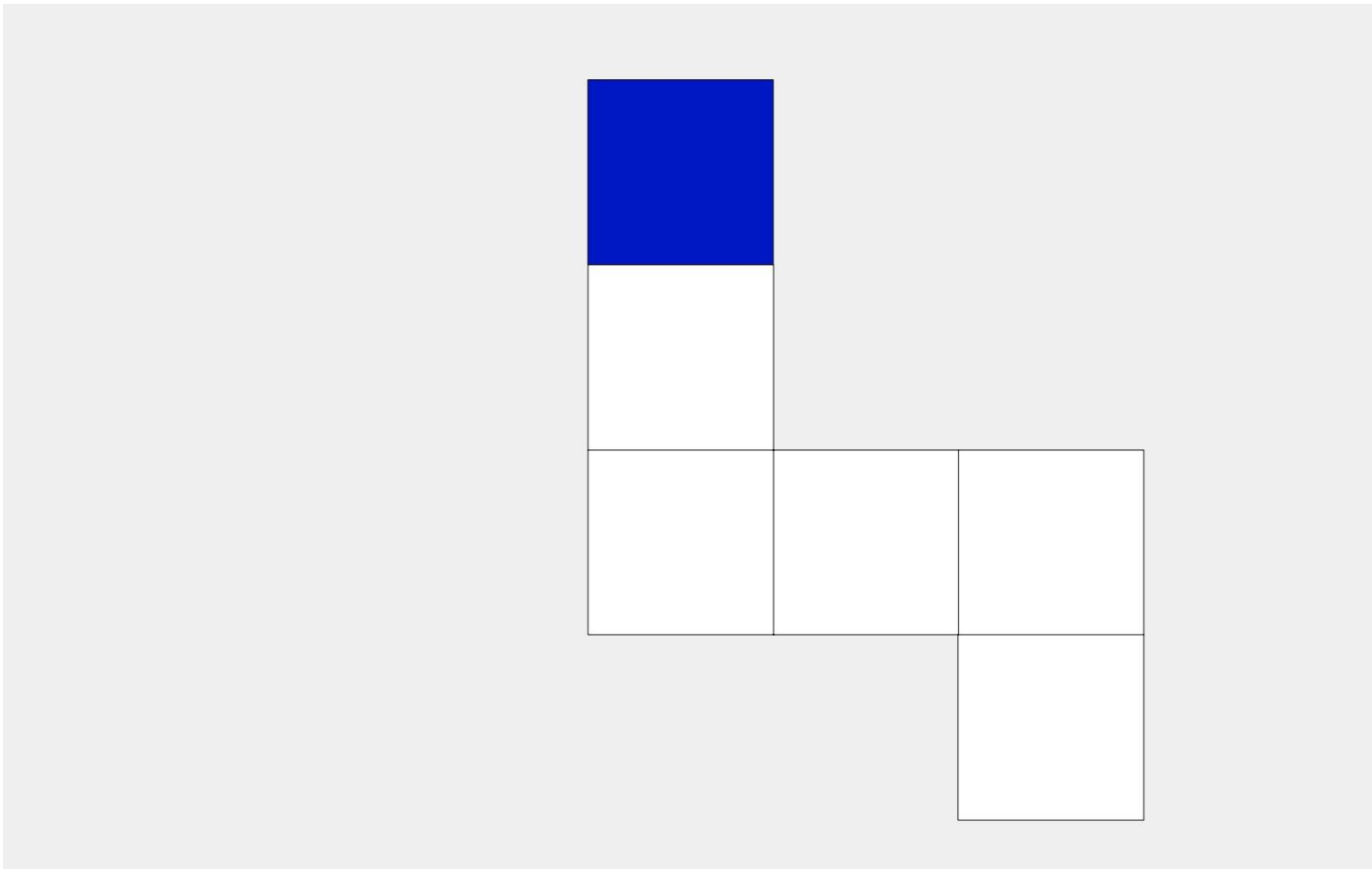
3 Software to develop spatial ability

Place the blue square so that you get a net of a cube. Then check if you can fold a cube.

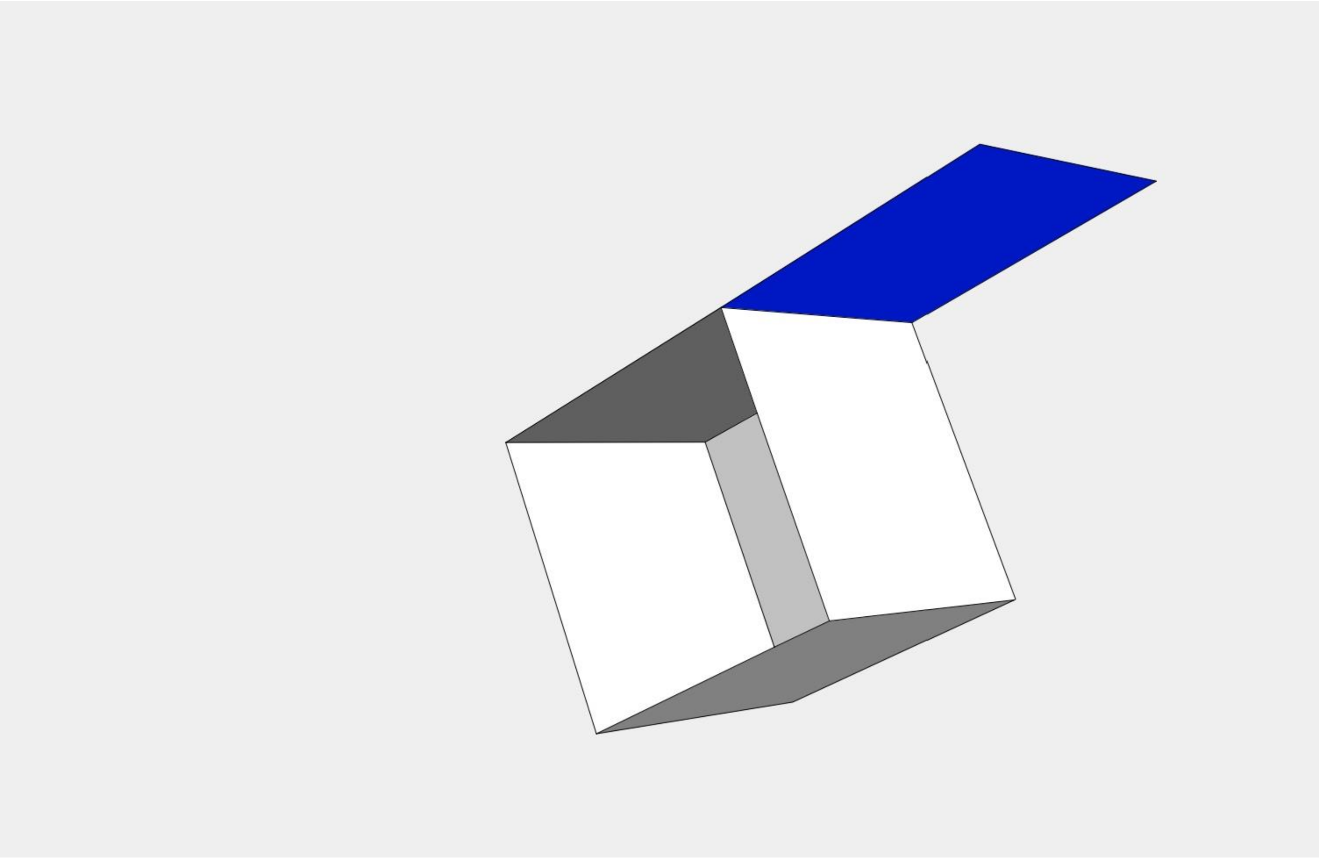




3 Software to develop spatial ability



Software to develop spatial ability



3 Software to develop spatial ability

3.2 possibilities

We hope, software can substitute a lot of manipulations with real 3D-manipulatives by ***virtual manipulatives***.

With software, many students have access to a visualization.

With the software, students can get an informative feedback just in time.

When students make a mistake, they can correct and try a better solution without spending a lot of time and material.

4

MATHEMATIKUS

Mathematikus.de is a website

- founded by me,
- designed by my daughter Doreen,
- further developed in further 2020/2021 as part of the iTEM project.
- iTEM is a common research project of TU Liberec and NORD University Bodø.

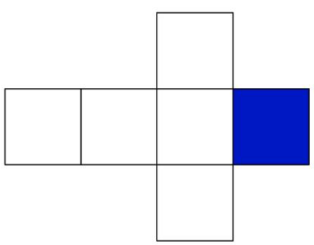
Using this website students can fold and solve tasks that require spatial imagination.

MATHEMATIKUS

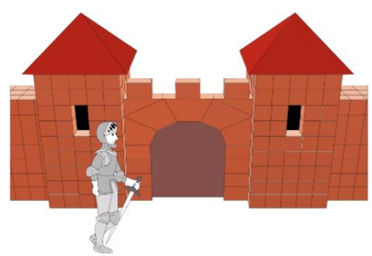
Úkoly pro rozvoj prostorové představivosti

Krychle, sítě krychlí a mnoho dalších úloh

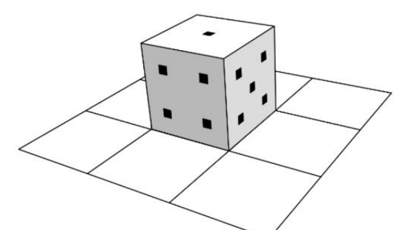
Zde najdete úlohy s krychlemi, sítěmi krychlí, hrady a dalšími zajímavými objekty. Při řešení každé úlohy si musíte něco představit. Vyberte si úlohu a vyzkoušejte ji.



Sítě krychle



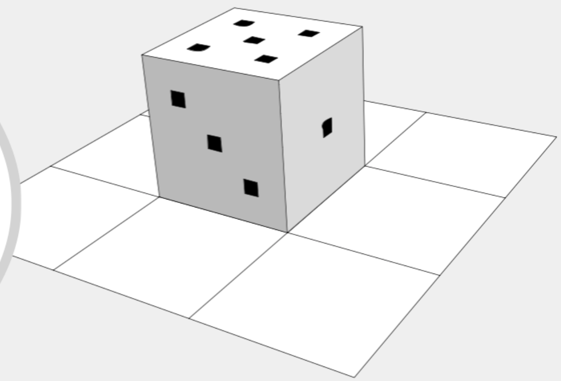
Rytíř Runkel

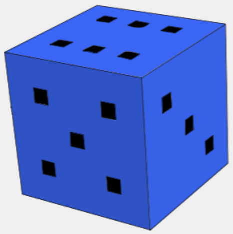


Odvalující se kostka

Odvalující se kostka – úloha 3

Odvalte bílou kostku ve 2 krocích tak, aby ležela ve stejné poloze jako modrá kostka.





Nejprve odvalte kostku dopředu, potom doleva.

Nejprve odvalte kostku doprava, potom dopředu.

Nejprve odvalte kostku dopředu, potom doprava.

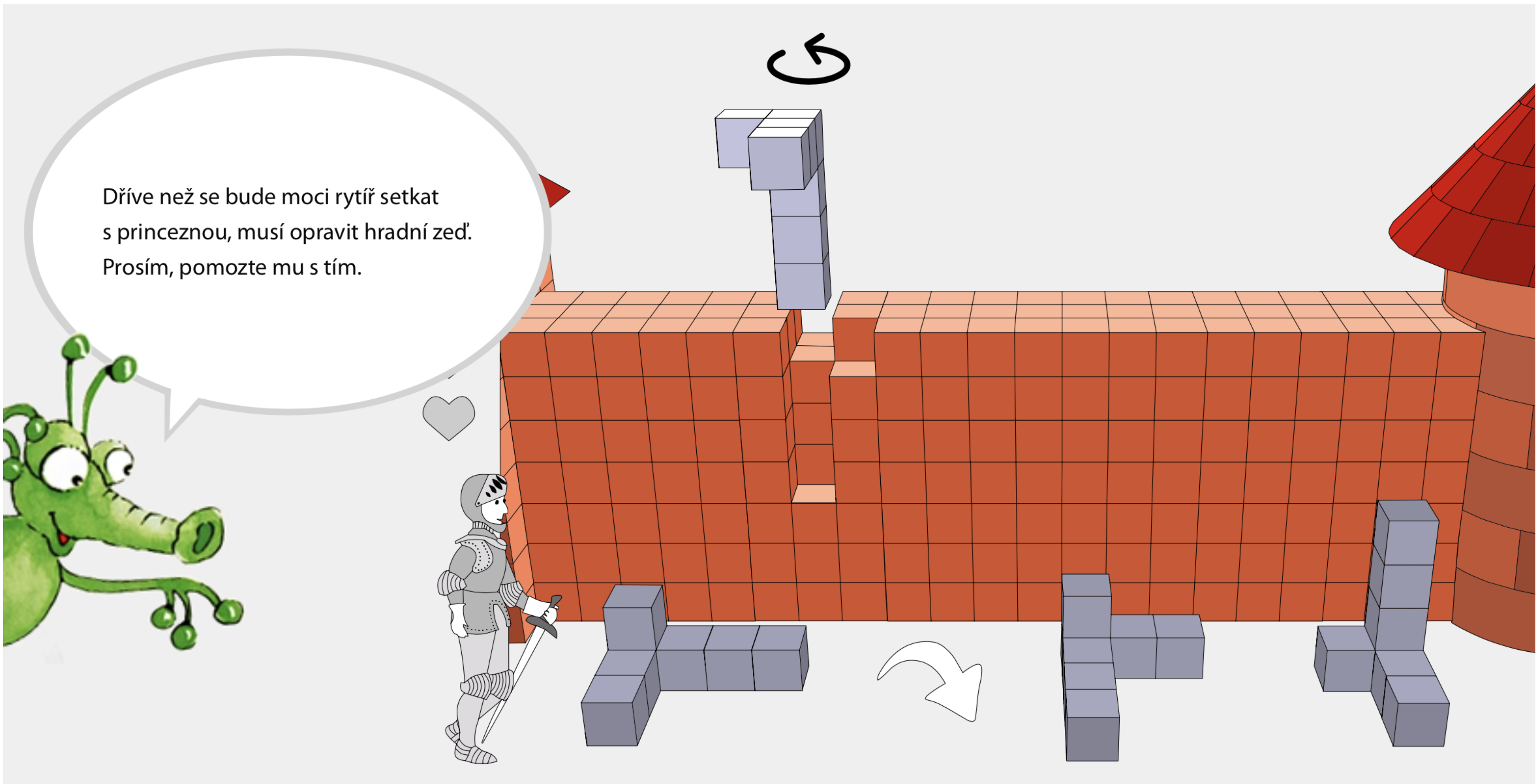
Nejprve odvalte kostku dozadu, potom doprava.

Nejprve odvalte kostku, doleva, potom dopředu.

To nefunguje.

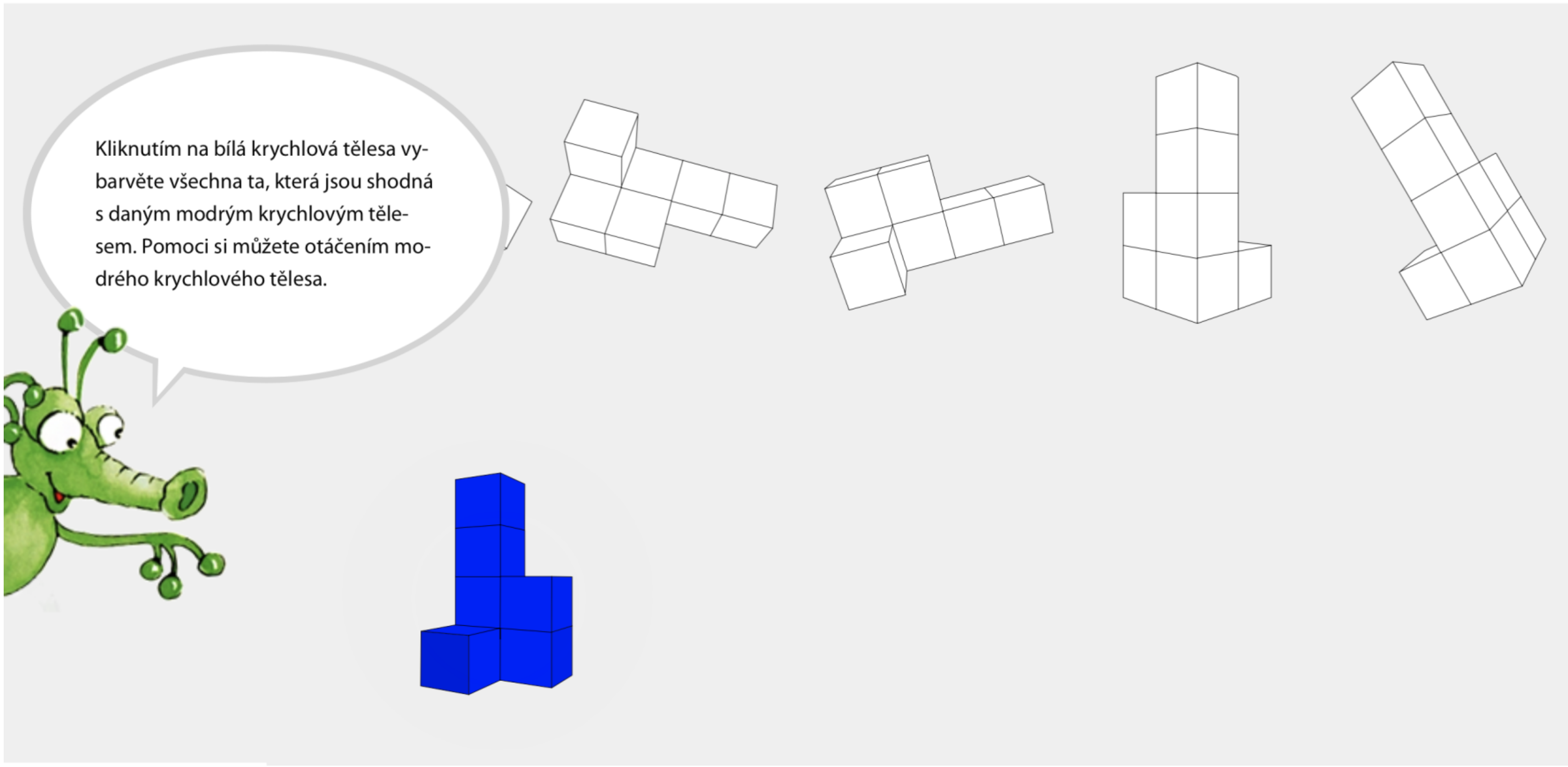
4 Mathematatikus: Imagination of rotation

Rytíř Runkel se chce setkat s princeznou.



Mathematatikus: Imagination of rotation

Rotující tělesa – úloha 1



5 Open questions – our research

There is one Problem

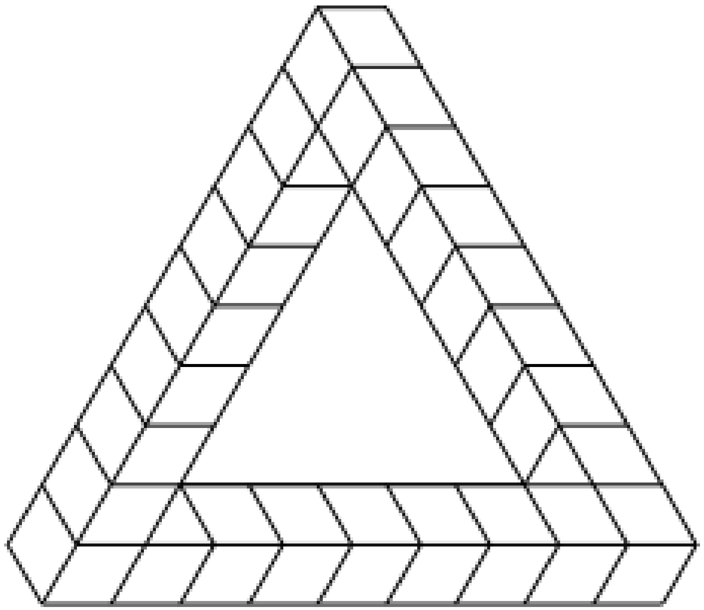
- Our real world is 3D, but the software runs on a screen in 2D.
- Key question: What experiences with 3D reality are necessary to allow students to solve spatial geometric tasks on the computer?
- On the screen as in reality, horizontal separations (bottom-up) and vertical separations (left-right) can be perceived in the same way.

5 Open questions – our research

Because software runs on a screen in 2D:

- Separations by planes perpendicular to the sagittal axis, i.e., by “depth planes”, we can perceive in the reality but not on the screen.
- Therefore, software always requires that students ***imagine*** spatial objects, spatial relations and spatial processes.

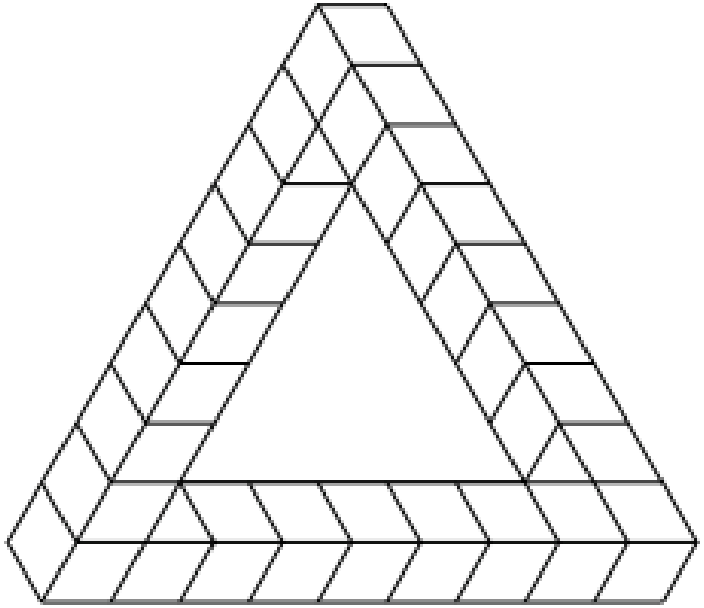
5 3D reality and 2D-pictures



The PENROSE-Triangle

2-D images are flat, they have no depth planes!

5 3D-reality and 2D-pictures



- Students should first use plastic cubes and try to build a Penrose triangle, as shown in this picture.

5 3D-reality and 2D-pictures

- Then they can close one eye and look from different perspectives at the wooden beam construction.
- They will discover that from one position, it looks like the Penrose triangle.

5

3D reality and 2D pictures

This is the 3D reality.

There is no triangle

There is a gap 3D-2D

5

Research activity MATHEMATIKUS

What we like to investigate

Students’ ways of working. How students arguing, describing etc.

- In detail, the question of whether the modules are usable, and students have the possibility to work effectively
- In detail, the question of whether the software encourages solving by trial and error
- In detail, differences in students’ reasoning between tasks given in 2D on the screen and in 3D in reality

MATHEMATIKUS

Try MATHEMATIKUS first for yourself, then see what your students do with it.

On behalf of the whole project team:

I wish you and your students

a lot of fun and success

working with MATHEMATIKUS.