Software-supported development of the ability to rotate mentally

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The importance of visuospatial abilities is undisputed. One crucial visuospatial qualification is the ability to rotate mentally. Studies have shown that it is possible to foster it through focussed training. Traditional tasks for training this ability require verifying the solution in the three-dimensional reality. This kind of control is often very time-consuming or not possible at all. Software offers a possible solution to this problem. This paper compares the working patterns of students on conventionally presented tasks with the working patterns on tasks presented by the software.

Keywords: Visuospatial ability, mental rotation, educational software, web-based training.

Problem

The importance of visuospatial abilities is undisputed and well documented (e.g., Battista, 1999; Maier, 1994). Research suggests that training can strengthen visuospatial abilities (Uttal et al., 2013). One important visuospatial ability is the ability to rotate mentally

When fostering visuospatial abilities, multiple obstacles can arise from conventional teaching practices: The availability of training materials is usually limited and restricted to widespread solids without atypical variations. Often, commonly used exercises contain time-consuming secondary activities that do not support the development of visuospatial abilities. If students like to check whether two depicted cube snakes have the same orientation in a task like the one depicted in Figure 1, at least one of them must be assembled and moved in space. Tasks like the ones shown in Figure 2, which require the student's imagination, are missing concrete three-dimensional verification methods. Therefore, students can be limited to their teacher's assessment without the possibility of visually comprehending the proposed solution.

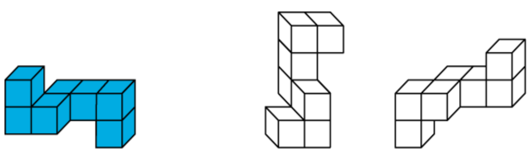


Figure 1: Cube snakes

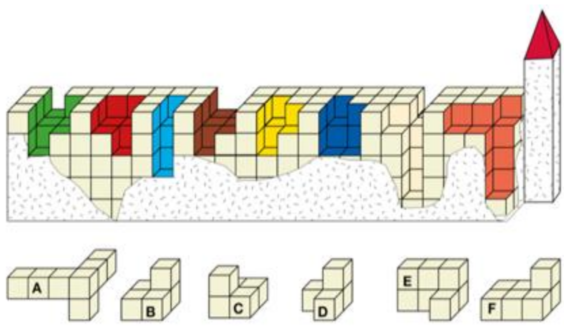


Figure 2: Which chunk fits into which gap?

Suitable software can help to overcome these obstacles. The possibilities of using software to develop visuospatial abilities seem promising: Many planar figures and solids can be generated comparatively effortlessly. It is possible to implement immediate visual feedback, such as observable animations.

Existing studies on the use of software refer to rotations in the plane (e.g., Frick, Ferrara, & Newcombe, 2013) often using statically given tasks without real informative feedback (e.g., Moen, Beck, Saltzmann, et al., 2020).

A potential limitation of software-supported development of visuospatial abilities is the two-dimensionality of computer screens. Since depth is not displayable on a computer screen, it seems questionable how far it is possible to promote visuospatial abilities using software. The presented study contributes to answering this question.

Theoretical background

"Imagination of rotation" as one of the visuospatial abilities

Visuospatial qualification is the ability of humans to perceive objects in their environment and mentally process these sensory impressions. We can create mental pictures of objects without regard for their actual existence and perform mental operations on them, such as mental rotation or spatial perspective-taking (cf. Maier 1994). This complex construct needs to be differentiated into multiple facets or aspects for which concrete tasks can be developed to enable efficient teaching. Numerous factor-analytic attempts have identified such composing factors (cf. Carroll 1993). Gutiérrez's (1996, p. 4) statement, "There is no general agreement about the terminology to be used in this field." still holds today.

Even though there is no clear consensus about the factor-analytical structure of visuospatial ability (cf. Gutiérrez, 1996; Maier, 1994), for our purposes, we utilised the following five reappearing aspects to categorise exercises that are meant to foster these abilities: Spatial perception, spatial visualisation, the imagination of rotation, the imagination of spatial relations, and spatial orientation. We focused on the development of the imagination of rotation. "Imagination of rotation" describes the mental ability to rotate a plane or solid figure quickly. The imagined rotation is around the horizontal, vertical, or sagittal axes.

The importance of visuospatial abilities

The pivotal role of visuospatial abilities in multiple scientific disciplines and engineering branches has been widely acknowledged (e.g., Mulligan, 2015; Wai et al., 2009). Visuospatial abilities enable us to perceive objects, the relationships between these objects, and spatial processes. The concept visuospatial abilities includes imagination and mental manipulation of these objects (e.g., Maier, 1994; Battista, 1999). These abilities are essential because we live in a three-dimensional world. Every interaction with the world around us is always a confrontation with its spatiality: descriptions of spatial objects, relationships, and processes are, with few exceptions, possible only through planar images or verbal representations. Therefore, any communication about three-dimensional objects, relationships, and processes always requires imaginative performances, namely the translation of our three-dimensional reality into a two-dimensional representation or a verbal description and vice versa.

Visuospatial abilities play a crucial role in many aspects of thinking, and their targeted improvement is within the realm of possibility (e.g., Gilligan, Thomas, & Farran, 2019). The ability to imagine numbers, relations between numbers, and operations, is crucial for success in mathematics (cf. Georges, Cornu & Schiltz, 2019). Neuroimaging studies support this idea: "Therefore, the parietal mechanisms that are thought to support spatial transformations might also be ideally suited to supporting arithmetic transformations” (Hubbard, Piazza et al. 2005, p. 445). Acquiring visuospatial ability is by no means merely useful for correctly answering a handful of scholastic geometry-related problems but instead seems to be of fundamental value in a myriad of areas, including scientific thinking, counting abilities, and arithmetic capabilities (Georges, Cornu & Schiltz, 2019).

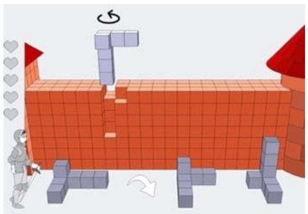
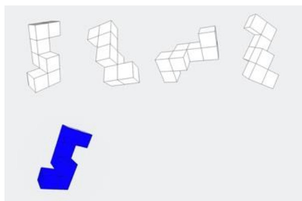
The role of software in the development of visuospatial abilities

For many years, high-performance hardware and fast internet connections are widespread and affordable. These allow for real-time rendering of complex three-dimensional scenes and processes. Web browsers allow complex transformations without violating physical laws. The accuracy of the spatial representation is ensured by the real-time conversion of these three-dimensional objects into a two-dimensional image for the computer screen.

The website mathematikus.de, which we developed offers various modules that promote the development of visuospatial abilities through exploratory methods. All modules focus on imagining objects and processes while saving time on side activities such as drawing, cutting, and folding, and extensive explanations. All modules are designed so users can understand the task immediately; an animation supplements each task as informative visual feedback (Eichler, 2008). Consequently, users can comprehend whether and why their solution is correct or incorrect. Finally, the user can control the software with a touch screen and does not have to learn keyboard shortcuts to control the software.

Our study focused on two modules of this website. Both modules contain tasks that are designed to foster mental rotation.

Table 1: Two Modules and their didactical background

Repair the castle wall The knight must repair the castle wall, which is missing parts. To fill each gap in the wall, the knight has a choice of several chunks composed of cubes. The selected chunk moves to the gap, and the student can rotate it. Thus, it is immediately apparent whether the chunk was selected correctly.	
Cube snakes (like Shepard & Metzler 1971) Given are images of differently oriented colourless figures consisting of cubes. The student should mark all colourless solids identical to the blue one. If he has problems mentally rotating the cubic solid, he can rotate the blue solid.	

Activities and the development of abilities

The development of abilities depends on specific activities; and performing the same activities must lead to the development of similar abilities. This fundamental principle of experience-dependent plasticity is a topic of ongoing investigation in neuroscience research (e.g. Ganguly & Poo, 2013). We can assume that the activities in analogue and digital learning environments have comparable effects if the students show similar working patterns and the relative frequency of the working patterns is similar across the whole set of problems.

Method

The study was conducted with 36 randomly selected third and fourth-grade students from two schools. To assess the software's potential for promoting visuospatial abilities, we compared the working patterns of two groups of students. One group of students worked on tasks with virtual 3D objects presented by the software (Table 1). The other group of students performed analogous tasks presented in three-dimensional reality (Figures 3 and 4).

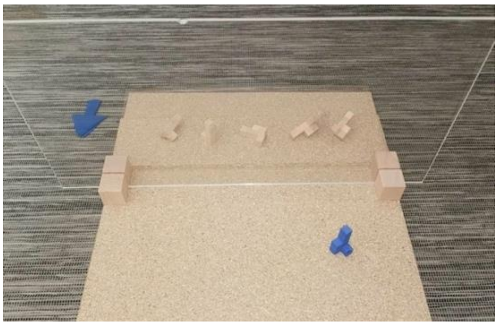


Figure 3: Cube snakes

Figure 4: Which chunk fits into the gap?

In both cases, the students worked in pairs and were asked to decide on a common solution. We gained insights into the students' way of thinking from the dialogues between the students and their gestures. All work was videotaped and transcribed. The transcripts were the starting point for developing a theory-based category system (cf. Mayring 2016, p. 114) and were analysed qualitatively using these categories. The qualitative content analysis aimed to identify the strategies used by the students to solve the tasks (cf. Mayring, 2016, p. 115). For the evaluation, "reasoning patterns" were created (cf. Mayring, 2016, p. 100). The analysis focused on the students' reasoning patterns and the verbal expressions and gestures characterising these reasoning patterns.

Findings

Strategies and reasoning patterns

The following strategies and reasoning patterns were found by analysing the students' verbal utterances and gestures while solving the tasks.

Strategy 1 is **rotating**. Students imagine the complete object(s), pay attention to the spatial relationships between the objects, and mentally manipulate one or both objects. We found three reasoning patterns by which we identified this strategy:

- Students use twisting and tilting movements of their hands or fingers to illustrate the imagination of a rotation (**Rotating pattern 1** – **RP1**).
- Students describe the movement of the cube snake or the chunk using typical keywords such as "turn" and "lay down" (**RP2**): "You have to turn it with your eyes, and then it fits."
- Students describe which part of the object is moved where and how (**RP3**): "This part goes here, and that part goes there."
- Students identify and name objects that are mirror images of each other based on their different orientations and justify the different orientations by the impossibility of rotating one object to lie like the other. (**RP4**).

Strategy 2 is **coding**. The student compares two objects by comparing their visual information, which he or she has fully internally encoded. The student focuses on parts of the visual information.

- Students count the cubes and describe their position using their fingers to count. Often synchronous pointing movements support statements such as “There are three up here, one on the right, and then three more here.” (**Coding pattern 1 – CP1**)
- Students make statements that indicate the comparison of the size of the cube snakes or the chunks (**CP2**): “This one … (Points to cube snake 3) … no, it is too short.”
- Students make statements about missing cubes to exclude certain cube snakes or chunks as solutions. “This one is also wrong (Points to cube snake 3). Because there is again a cube missing.” (**CP3**)
- Students compare single segments of a cube snake or a chunk of several cubes with another cube snake or a gap in the wall. “No, this one (is correct). (Points to a blue chunk.) Because here is again such a box and here is again one cube missing.” (**CP4**)

Strategy 3 is **mirroring**. Students mirror an object mentally or recognise that an object (a cube snake or a chunk) is the spatial mirror image of another object.

- Explicitly naming an object as the mirror object of another one or capturing the object mirroring, but only paraphrasing it, for instance: “… it is exactly the other way around.” Or “…it is reversed once…” (**Mirroring pattern 1 – MP1**).
- Transitive reasoning about the orientation of an object, such as, “… if that is the mirror object of the first object, and this is the mirrored version of that one, then …” (**MP2**).

Strategy 4 is **trying by object manipulation**. We identified it only in the task “cube snake.”

- Students try to manipulate a cube snake so that it looks like the other one is. They move the blue cube snake in different positions and compare: “Look, it is exactly the same.” (**TP1**)

Strategy 5 is **Guessing by interpreting feedback**. Based on an informative feedback, students guess a solution and modify it if necessary.

- Students try it out, receive informative feedback, and use that feedback to solve the task. “Let’s try this … (selects, receives feedback) … no, that is not right, then this must be correct.” (**Guessing pattern 1 - GP1**).
- The students verbalise that they are trying systematically without any spatial imagination. Feedback tells them how many solutions are still missing. (Feedback: “2 out of 3 cube snakes found.”). “There can be only one correct left.” (He keeps trying) (**GP2**).

Only the strategies and corresponding reasoning patterns described above, were found in the present study. Other strategies like “surface thinking,” “verifying strategy,” and “falsifying strategy” described by Dünser (2005, p. 88) or Maresch (2015, p. 136 ff.) could not be identified. In our opinion, the reason is, that this results from the characteristics of the tasks we used. In both presentation modes, both tasks give students almost no reason or opportunity to choose these strategies.

Comparison of reasoning patterns when working on real 3D tasks and tasks posed by software

The following table shows the usage of the described reasoning pattern. For each reasoning pattern, it is compared how often it was used by the students when solving the task given in 3D and how often it was used when solving the task presented by the software. Note that the number of uses refers not to a division among students, but rather to the actual number of uses of reasoning patterns. Since the module "cube snakes" contains four tasks and the module "repair the castle wall" contains five tasks each pair of students could worked on a maximum of 9 tasks. We analyzed the videos, specifically the students' verbal utterances and gestures, and recorded the strategy used by every student for each task worked on.

When working on the task, occasionally, both students of a pair found the same solution using different strategies. It also happened that one student applied several strategies. That is why the sums in Table 2 are other than the number of students.

Table 2: Number of uses of the reasoning patterns

		cube snakes		**repair the castle wall**	
strategy	**reasoning pattern**	**real 3D**	**virtual 3D**	**real 3D**	**virtual 3D**
rotating	RP1	5 (3.5%)	0	3 (5.7%)	6 (12.2%)
	RP2	39 (27.3%)	45 (27.8%)	11 (19.6%)	10 (20.4%)
	RP3	5 (3.5%)	2 (1.2%)	0	0
	RP4	17 (11.9%)	1 (0.6%)	0	0
sum (rotating)		66 (46.2%)	48 (29.6%)	14 (25.0%)	10 (20.4%)
coding	CP1	21 (14.7%)	30 (18.5%)	28 (50.0%)	13 (26.5%)
	CP2	0	1 (0.6%)	3 (5.4%)	2 (4.1%)
	CP3	3 (2.1%)	0	1 (1.8%)	1 (2.0%)
	CP4	8 (5.6%)	6 (3.7%)	9 (16.1%)	16 (32.6%)
sum (coding)		32 (22.4%)	37 (22.8%)	41 (73.2%)	32 (65.3%)
mirroring	MP1	2 (1.4%)	2 (1.2%)	1 (1.8%)	1 (2.0%)
	MP2	3 (2.1%)	1 (0.6%)	0	0
sum (mirroring)		5 (3.5%)	3 (1.8%)	1 (1.8%)	1 (2.0%)
trying	TP1	38 (26.6%)	53 (32.7%)	0	5 (10.2%)

Discussion

The results show that regardless of the mode of presentation of the tasks, the observed students used almost identical patterns of reasoning with similar frequency, except for minor deviations. We can conclude from the recorded reasoning patterns the strategies used to solve the tasks. The data show that students used the same reasoning patterns when working on tasks presented by the software as they did for tasks presented in 3D reality and that the relative frequency of occurrence of these reasoning patterns is also similar in both forms of presentation.

These facts strengthen our assumption that students can similarly improve their spatial-visual abilities when using appropriate software comparable to working on tasks presented with 3D objects. This statement is firstly relative because the number of students involved was relatively small, n=36. The statement is secondly limited to only one component, the ability to rotate mentally. Third, the statement is limited to students in grades 3 and 4 and above. These students obviously have sufficient prior experience to grasp the tasks on the two-dimensional screen. It needs to be investigated to what extent similar results occur for students in grades 1 and 2. Last but not least, the promising outcome of this study encourages us to extend the development and research of the platform Mathematikus.de.

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References

Battista, M. T. (1999). The importance of spatial structuring in geometric reasoning. *Teaching children mathematics*, *6*(3), 170–177. http://doi.org/10.5951/TCM.6.3.0170

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. Cambridge University Press. http://doi.org/10.1017/CBO9780511571312

Cheng, Y.-L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, *15*(1), 2–11. http://doi.org/10.1080/15248372.2012.725186

Cornu, V., Hornung, C., Schiltz, C., & Martin, R. (2017). How do different aspects of spatial abilities relate to early arithmetic and number line estimation? *Journal of Numerical Cognition*, *3*(2), 309–343. http://doi.org/10.5964/jnc.v3i2.36

Dünser, A. (2005). *Trainierbarkeit der Raumvorstellung mit Augmented Reality*. [dissertation].Universität Wien, Fakultät für Psychologie.

Eichler, D. (2008). *Konzeption und Realisierung einer Software zur Entwicklung des räumlichen Vorstellungsvermögens*. [unpublished Diplomarbeit.] Universität Ulm.

Frick, A., Ferrara, K., & Newcombe, N. S. (2013). Using a touch screen paradigm to assess the development of mental rotation between 3½ and 5½ years of age. *Cognitive Process*, *14*(2), 117–127. http://doi.org/10.1007/s10339-012-0534-0

Ganguly, K., & Poo, M. (2013). Activity-dependent neural plasticity from bench to bedside. *Neuron*, *80*(3), 729–741. http://doi.org/10.1016/j.neuron.2013.10.028

Georges, C., Cornu, V., & Schiltz, C. (2019). Spatial skills first: The importance of mental rotation for arithmetic skill acquisition. *Journal of Numerical Cognition, 5*(1), 5–23. http://doi.org/10.5964/jnc.v5i1.165

Gilligan, K. A., Thomas, M. S. C., & Farran, E. K. (2020). First demonstration of effective spatial training for near transfer to spatial performance and far transfer to a range of mathematics skills at 8 years. *Developmental Science, 23*(4). http://doi.org/10.1111/desc.12909

Gutiérrez, A. (1996). Vizualization in 3-dimensional geometry: search of a framework. In L. Puig and A. Guttierez (Eds.) *Proceedings of the 20th conference of the international group for the Psychology of Mathematics Education* (Vol. 1, pp. 3–19). Universidad de Valencia.

Hubbard E. M., Piazza, M., Pinel P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience, 6*, 435–448. http://doi.org/10.1038/nrn1684

Maier, P. H. (1994). *Räumliches Vorstellungsvermögen - Komponenten, geschlechtsspezifische Differenzen, Relevanz, Entwicklung und Realisierung in der Realschule*. Peter Lang

Maresch, G. (2015). How to develop spatial ability? Factors, strategies, and gender specific findings. *Journal for Geometry and Graphics, 19*(1), 133–157.

Mayring, P. (2016). *Einführung in die qualitative Sozialforschung*. Beltz.

Moen, K. C., Beck, M. R., Saltzmann, S. M., Cowan, T. M., Burleigh, L. M., Butler, L. G., Ramanujam, J., Cohen, A. S., & Greening, S. G. (2020). Strengthening spatial reasoning: elucidating the attentional and neural mechanisms associated with mental rotation skill development. *Cognitive Research: Principles and Implications, 5*(1), 20. http://doi.org/10.1186/s41235-020-00211-y

Mulligan, J. (2015). Looking within and beyond the geometry curriculum: connecting spatial reasoning to mathematics learning. *ZDM – Mathematics Education, 47*(3), 511–517. http://doi.org/10.1007/s11858-015-0696-1

Newcombe, N. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator, 34*(2), 29–36.

Shepard, R. N. & Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science, 171*(3972), 701–703. http://doi.org/10.1126/science.171.3972.701

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial abilities: a meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352–402. http://doi.org/10.1037/a0028446

Wai, J., Lubinski, D., & Benbow, C. (2009, 11/01). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, *101*(4), 817–835. http://doi.org/10.1037/a0016127