

TU Liberec

Geoworkshop – Part 01

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NORD
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Purpose of teaching geometry - material

- We live in a geometrical world.
- Our interaction and confrontation with this world is also an exploration of its geometrical properties.
- Everyday problems are often an occasion for geometrical activities and investigations.

1 Purpose of teaching geometry - formal

Harmonious development of the child:

- *arousing **curiosity and interest** in mathematical activities, objects and problems,*
- *waking and developing **pleasure and interest** in mathematical activities specially and pleasure in learning generally,*
- *promoting imagination and creativity,*

1 Purpose of teaching geometry

geometrical knowledge allows to understand our world
Bridge constructions – the function of the triangles.



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Purpose of teaching geometry

Geometrical knowledge allows to understand and reconstruct our world:

Analyze bridge constructions.

Then, using 20 sheets of paper and some glue, build a bridge that is as stable as possible.



1 Purpose of teaching geometry

Geometrical knowledge allows to understand and reconstruct our world:

Analyze bridge constructions.

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Aspects of Geometry (E. Ch. Wittmann)

1. geometrical objects (surfaces and bodies) and their properties
2. operations with shapes and bodies
3. relationships between geometry and reality
4. reference systems for describing the world and working in geometry
5. measurement and quantities
6. regularities / logical and functional relationships
7. connections between geometry and arithmetic

1 Tasks and how to work suitable with them

Working with tasks as the teacher's activity means (cf. Fanghänel, 2000):

- select and arrange the tasks,
- instruct the students work with the tasks,
- start and keep running the process of solving the tasks, and
- initiate the reflection of the result and the way to this result.

Tasks and how to work suitable with them

- In daily work, a teacher is unable to prepare a lot of different tasks for a lot of different learners.
- How does the teacher know, which child is able to solve which task? Can he know every day which task fits which student?
- Every student will be frustrated, if he day by day gets only stupid tasks whilst another student gets interesting problems.

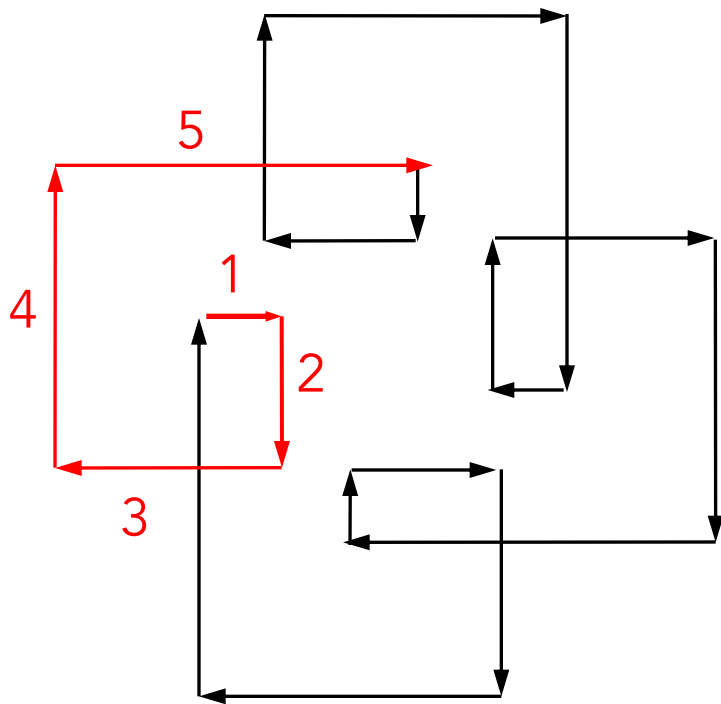
Tasks and how to work suitable with them

Providing „substantive“ tasks for all students:

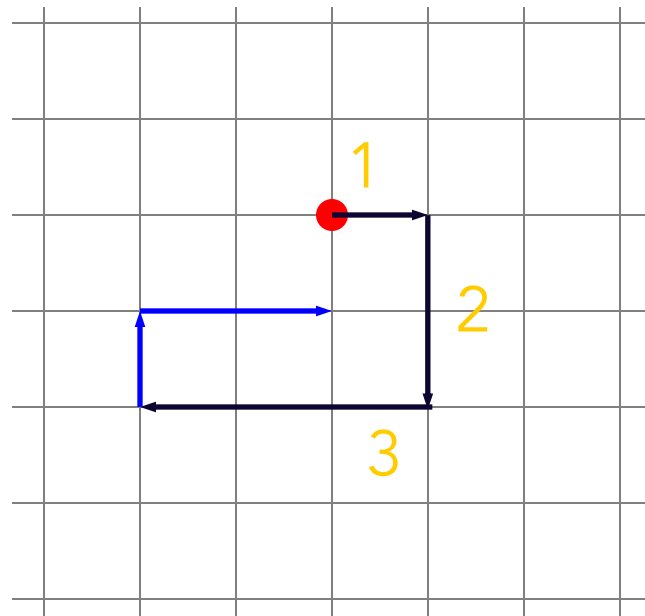
- All students can capture and describe the nature of these tasks.
- All students can achieve elementary results.
- The tasks offer opportunities for further work at higher levels.
- The tasks offer opportunities for deeper understanding, they challenge gifted students.

Example: Spirolaterals

1 - 2 - 3 - 4 - 5



Take 3 or 4 or 5 numbers ...
e.g., 1-2-3, 5-6-2, 2-4-1-2 ...



Example: Spirolaterals

firstly ...

- a simple task, a good exercise for drawing

secondly ...

- a possibility to explore,
- a possibility to assume facts, dependencies ...

thirdly ...

- a chance to swap the given object and the object in request

Example 1: Spirolaterals

Draw, find questions. Which questions are interesting? Which hypotheses are possible?

Think about, draw and find answers.

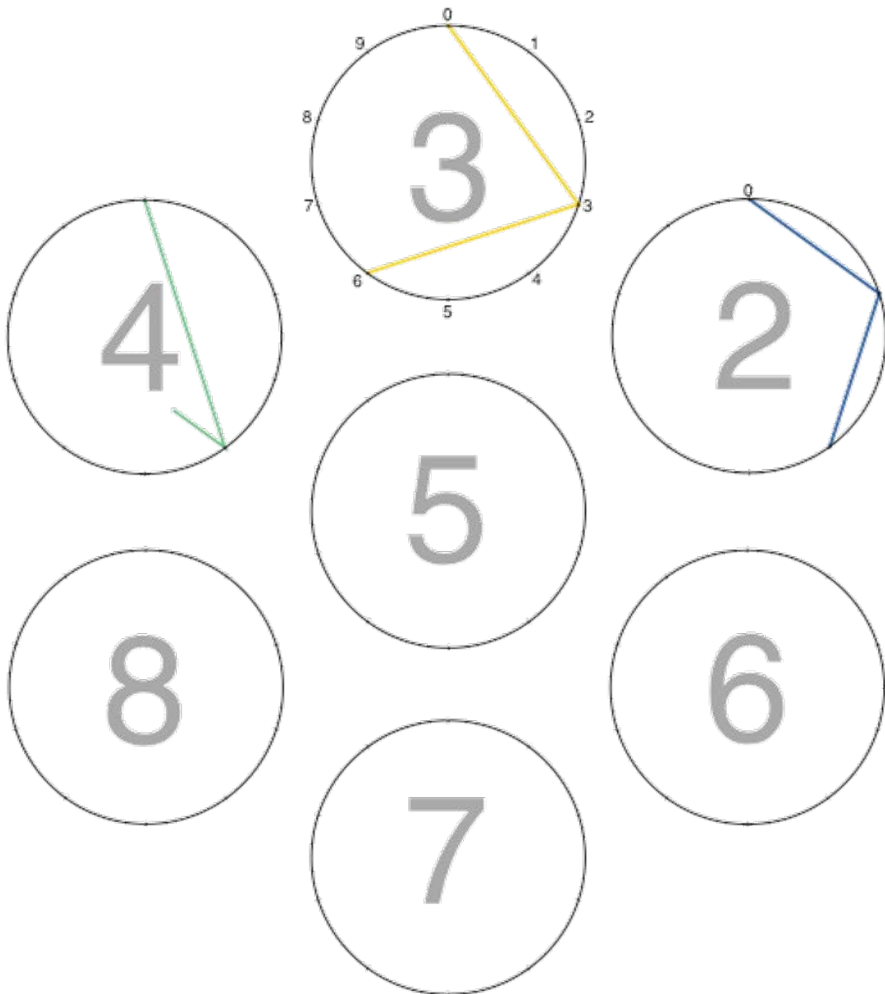
- Compare $1 - 2 - 3$ and $3 - 2 - 1$
- Compare $1 - 2 - 3$ and $2 - 4 - 6$
- Compare $2 - 3 - 7$, $2 - 3 - 8$ and $2 - 3 - 9 \dots$
- Choose $a - b - c - d$
- Choose $a - b - c - d - e$

Example: Spirolaterals a-b-c

right	down	left	upwards

How with 4 numbers, 5 numbers, 6 numbers ...

Example 2: Circles of ten



Example 2: Circles of ten

firstly ...

- a simple task, a good exercise for drawing

secondly ...

- a possibility to experiment,
- a possibility to assume facts, dependencies ...

thirdly ...

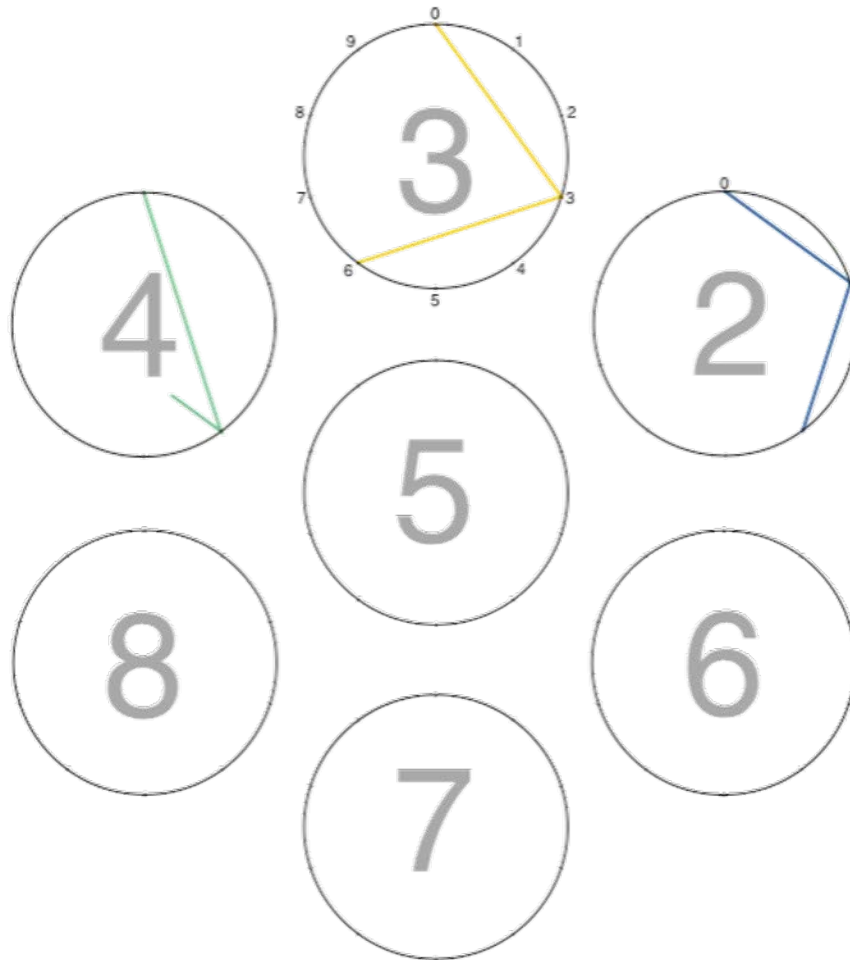
- a chance to swap the given object and the object in request

THE SAME QUESTIONS AS BEFORE!

Example 2: extentions

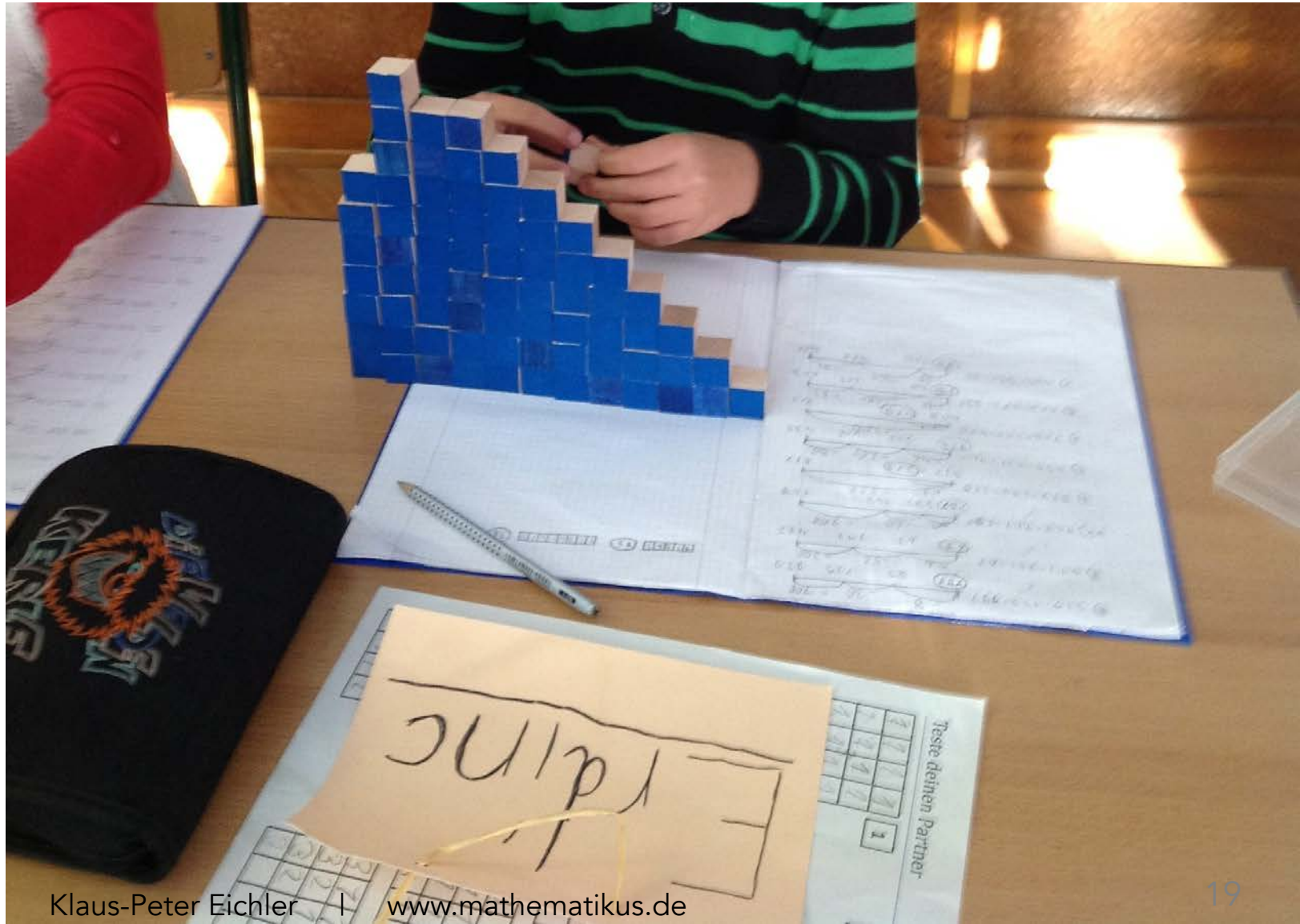
- Patterns on the clockface (twelve parts)
- Patterns in the calendar
- What happens if we multiply instead of add?
- ... Trigger the exploration of structures and after that come up with arithmetic ideas.

Example 2: Connection to arithmetics

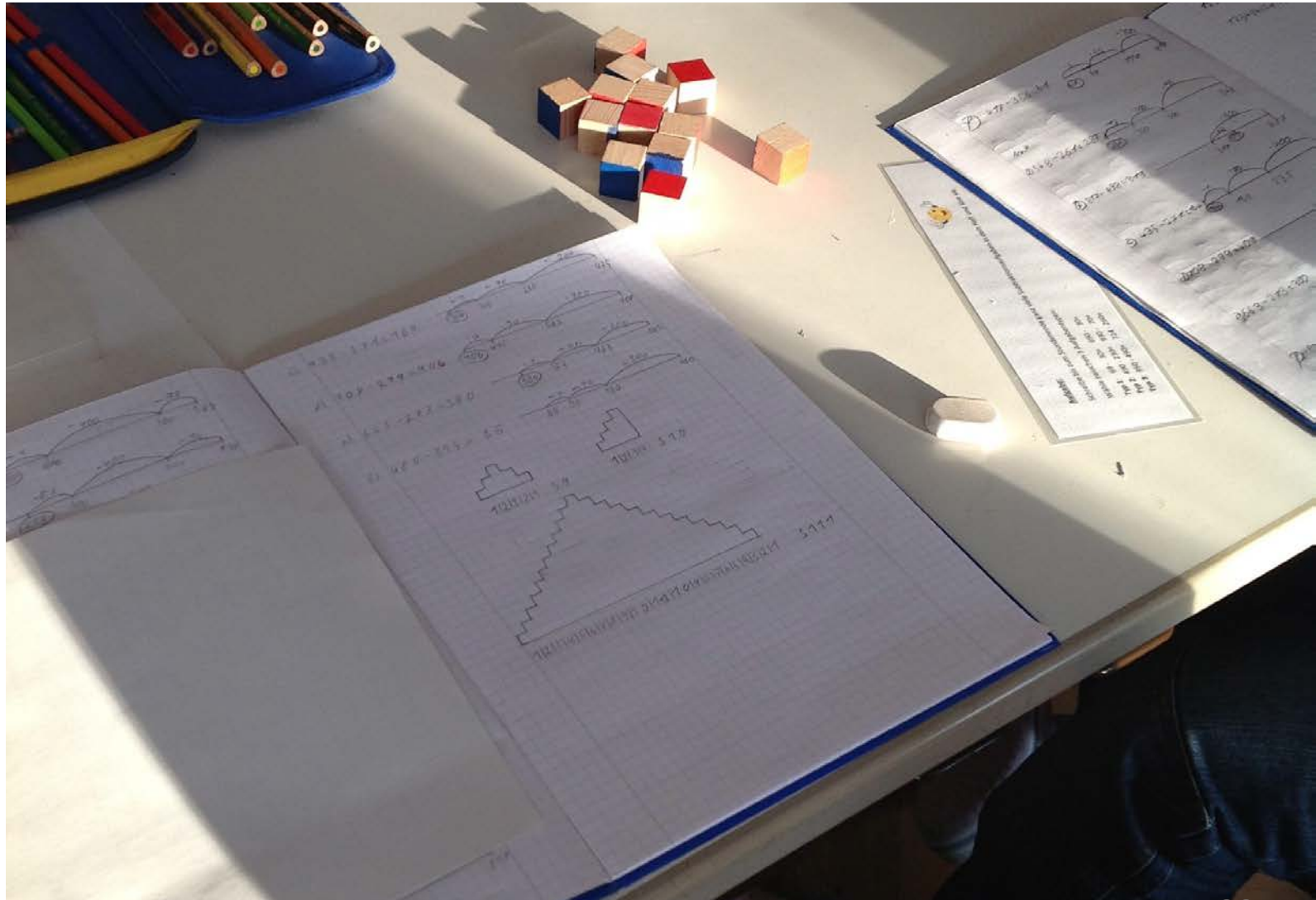


- Which stars touch all the numbers?
- What changes if you use 12 numbers?
- Is there a circle where all stars touch all numbers?

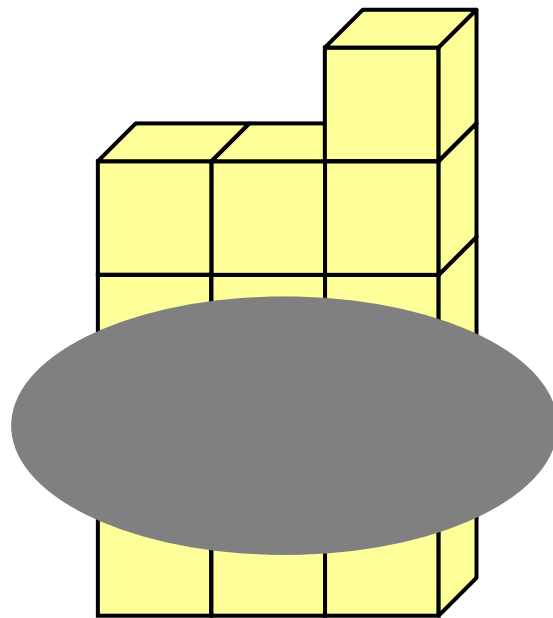
Example 3: Cube stairways



Cube stairways - plans



Cube stairways – tree steps



3	4	5
---	---	---

12

Cube stairways - variations

Different starting heights

5	7	9	11	13
---	---	---	----	----

45

6	8	10	12	14
---	---	----	----	----

50

7	9	11	13	15
---	---	----	----	----

55

Cube stairways - variations

Different height of the steps

5	7	9	11	13
---	---	---	----	----

45

5	8	11	14	17
---	---	----	----	----

55

5	9	13	17	21
---	---	----	----	----

65

Cube stairways - variations

swap the given object and the object in request

2	4	6	8	10
---	---	---	---	----

30

8	9	10	11	12
---	---	----	----	----

50

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Cube stairways – arithmetic context

Chip and Chap

Example 4: Calculation stairways

$$5 + 2 = 7$$

$$2 + 7 = 9$$

$$7 + 9 = 16$$

Example 4: Calculation stairways

firstly ...

- a simple task, a good exercise for calculation

secondly ...

- a possibility to experiment,
- a possibility to assume facts, dependencies ...

thirdly ...

- a chance to swap the given object and the object in request

Example 4: Calculation stairways

Swap the given object and the object in request:

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = 100$$

Example 4: Calculation stairways

A possibility to assume facts, dependencies ...

$$5' + 2 = 7'$$

$$2 + 7' = 9'$$

$$7' + 9' = 16''$$

Describe and **prove** dependencies

suitable work with good tasks

Are there solutions for every x ?

$$\begin{array}{ccccccc} _ & + & _ & = & _ & & \\ & & & & & & \\ & & _ & + & _ & = & _ \\ & & & & & & \\ & & & & _ & + & _ = x \end{array}$$

$$\begin{array}{ccccccc} a & + & b & = & a + b & & \\ & & b & + & a + b & = & a + 2b \\ & & & & a + b + a + 2b & = & 2a + 3b \end{array}$$

Using variable - Is that necessary?

Better: Think about the dependencies than stupid calculating

Finally: Solving tasks is finding patterns

if we are solving tasks

- we are always not only searching a solution
- we are always searching patterns
- trying patterns, transforming patterns
- keeping successful patterns...
- that's our live – we are looking for patterns



Thank you for your interest

It's time for discussion ...