Srdečně zveme pracovníky KMD, KAP a další zájemce z řad veřejnosti na přednášku

Approximate inverse preconditioning with adaptive dropping

Autoři: J. Kopal[1], M. Rozložník[2], M. Tůma[2]

[1] Technical University of Liberec, Liberec, [2] Institute of Computer Science, Academy of Sciences of the Czech Re., Prague

Přednáší: **Ing. Jiří KOPAL (KMD FP TU v Liberci)**

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Abstrakt přednášky:

The contribution deals with an approximate inverse preconditioning for the conjugate gradient method. The main goal is to compute the decomposition that would be sparse and reliable at the same time. In particular, the generalized Gram–Schmidt process with an adaptive dropping strategy is considered.

Assume the system of linear algebraic equations in the form $Ax = b$, where A is symmetric and positive definite. Symmetrically preconditioned system can be written in the form

$$\tilde{Z}^T A \tilde{Z} y = \tilde{Z}^T b, \qquad x = \tilde{Z} y,$$

where $\tilde{Z}$ is the factor of the approximation $\tilde{Z}\tilde{Z}^T$ to A^{-1}, that plays the role of the preconditioner.

The generalized Gram–Schmidt process in exact arithmetic provides matrices Z and U, so that $U^T U = (Z^{(0)})^T A Z^{(0)}$, $Z^T A Z = I$, and $ZU = Z^{(0)}$. The columns of the matrix $Z^{(0)}$ are initial vectors that are A-orthogonalized against previously computed vectors. Matrix U contains the orthogonalization coefficients. It is clear that U is the Cholesky factor of $A = U^T U$ for $Z^{(0)} = I$. In practice, the effects of the finite precision arithmetic should be considered. The corresponding bounds for the norms $\|\bar{Z}^T A \bar{Z} - I\|$, $\|\bar{Z}\bar{U} - I\|$, $\|\bar{U}^T \bar{U} - (Z^{(0)})^T A Z^{(0)}\|$ were derived in [2], where the approximate quantities (obtained by dropping) are denoted with an extra tilde.

The results in [2] motivate a new scheme where the accuracy of computed column vectors in $\tilde{Z}$ reflect the level of errors committed throughout previous factorization steps. Namely, the entries that have magnitudes smaller than the current level of the error can be dropped in any case. This dropping level is a function of the theoretical error bound for orthogonalization scheme. The whole scheme can be interpreted as a computation with the roundoff unit much larger than the standard $u = \epsilon/2$ [1]. Our implementation uses additional techniques that can significantly enhance numerical properties of the preconditioner and its sparsity, namely pivoting and diagonal scaling. In particular, these techniques support to achieve the main goal by approximate minimization of the conditioning of $\bar{U}$.

The theoretical results will be accompanied by carefully chosen experiments that demonstrate usefulness of the approach. We hope that the developed algorithm may extend scope of applicability of the considered type of approximate inverse preconditioners.

Literatura:

[1] J. Kopal, M. Rozložník, M. Tůma: *Approximate inverse preconditioners with adaptive dropping.* submitted to International Journal of Advances in Engineering Software, 2013.

[2] M. Rozložník, M. Tůma, A. Smoktunowicz, J. Kopal: *Numerical stability of orthogonalizationmethods with a non-standard inner product*. BIT Numerical Mathematics, 52 (4), pp. 1035-1058, 2012.

Za organizátory semináře srdečně zve

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