

Substituce ve dvojném a trojném integrálu

Příklad 1. Graficky znázorněte množinu M a vypočtěte:

a) $\iint_M x^2 + y^2 \, dx \, dy$, kde $M = \{[x, y] \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4, y \geq |x|\}$. $\left[\frac{15\pi}{8}\right]$

b) $\iint_M xy \, dx \, dy$, kde $M = \left\{[x, y] \in \mathbb{R}^2 : \frac{x^2}{4} + \frac{y^2}{25} \leq 1, x \geq 0, y \geq 0\right\}$. $\left[\frac{25}{2}\right]$

c) $\iint_M x \, dx \, dy$, kde $M = \{[x, y] \in \mathbb{R}^2 : (x-1)^2 + y^2 \leq 4\}$. $[4\pi]$

d) $\iint_M \frac{1}{1+x^2+y^2} \, dx \, dy$, kde $M = \{[x, y] \in \mathbb{R}^2 : x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$. $\left[\frac{\pi \ln 2}{4}\right]$

e) obsah obrazce $M = \{[x, y] \in \mathbb{R}^2 : 0 \leq x \leq y, 2y \leq x^2 + y^2 \leq 4\}$. $\left[\frac{\pi}{4} - \frac{1}{2}\right]$

f) $\iiint_M x^2 \, dx \, dy \, dz$, kde $M = \{[x, y, z] \in \mathbb{R}^3 : x^2 + z^2 \leq 4, y \in [0, 1]\}$. $[4\pi]$

g) objem tělesa $M = \{[x, y, z] \in \mathbb{R}^3 : 0 \leq z \leq 9 - x^2 - y^2\}$. $\left[\frac{81\pi}{2}\right]$

h) objem tělesa $M = \{[x, y, z] \in \mathbb{R}^3 : x^2 + z^2 + 1 \leq y \leq 5\}$. $[8\pi]$

i) $\iiint_M z^2 \, dx \, dy \, dz$, kde $M = \{[x, y, z] \in \mathbb{R}^3 : 4 \geq y^2 + z^2, x \in [0, 4]\}$. $[16\pi]$

j) $\iiint_M x^2 + y^2 \, dx \, dy \, dz$, kde $M = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 \leq 4, 0 \leq z \leq 2 - y\}$. $[16\pi]$

k) $\iiint_M x \, dx \, dy \, dz$, kde $M = \left\{[x, y, z] \in \mathbb{R}^3 : \frac{x^2}{4} + \frac{y^2}{9} \leq 1, z \in [0, 1]\right\}$. $[0]$

l) $\iiint_M y \, dx \, dy \, dz$, kde $M = \{[x, y, z] \in \mathbb{R}^3 : x^2 + (y-2)^2 \leq 4, z \in [0, 1]\}$. $[8\pi]$

m) těžiště homogenního tělesa $M = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4, z \geq 0\}$. $\left[0, 0, \frac{3}{4}\right]$