Cvičení 10

Příklad 1. Vypočtěte (také jako určité integrály)

- $\int 9x^6 + 6x^3 - 14\sin x\,dx \qquad \left(\overset{c}{=} \frac{9x^7}{7} + \frac{3x^4}{2} + 14\cos x \quad \text{pro } x \in \mathbb{R}\right)$,
- $\int \frac{(x-1)^2+1}{(x-1)^3}\,dx \quad \left(\overset{c}{=} \ln|x-1| - \frac{1}{2(x-1)^2} \quad \text{pro } x \in (-\infty,1) \text{ nebo } x \in (1,\infty)\right)$,
- $\int \frac{\sqrt[3]{x^2}}{4} + 3x^2\sqrt{x} - 2\cdot 5^x\,dx \qquad \left(\overset{c}{=} \frac{3x\sqrt[3]{x^2}}{20} + \frac{6}{7}x^3\sqrt{x} - \frac{2\cdot 5^x}{\ln 5} \quad \text{pro } x \in (0,\infty)\right)$,
- $\int \frac{x^5+x^4-3x^2-x}{x^2+1}\,dx \qquad \left(\overset{c}{=} \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} - 4x + 4\operatorname{arctg} x \quad \text{pro } x \in \mathbb{R}\right)$,
- $\int \frac{\cos 2x}{\sin^2 x\cos^2 x}\,dx \qquad \left(\overset{c}{=} -\operatorname{cotg} x - \operatorname{tg} x \quad \text{pro } x \in \left(\frac{k\pi}{2}, \frac{(k+1)\pi}{2}\right),\ k \in \mathbb{Z}\right)$,
- $\int \operatorname{cotg}^2 x\,dx \qquad \left(\overset{c}{=} -\operatorname{cotg} x - x \quad \text{pro } x \in (k\pi, (k+1)\pi),\ k \in \mathbb{Z}\right)$,
- $\int (2x-1)e^{2x}\,dx \qquad \left(\overset{c}{=} e^{2x}(x-1) \quad \text{pro } x \in \mathbb{R}\right)$,
- $\int x^2\ln(2x)\,dx \qquad \left(\overset{c}{=} \frac{x^3\ln(2x)}{3} - \frac{x^3}{9} \quad \text{pro } x \in (0,\infty)\right)$,
- $\int x\operatorname{arctg} x\,dx \qquad \left(\overset{c}{=} \frac{1}{2}\left(x^2\operatorname{arctg} x + \operatorname{arctg} x - x\right) \quad \text{pro } x \in \mathbb{R}\right)$,
- $\int \ln(x^2+1)\,dx \qquad \left(\overset{c}{=} x\ln(x^2+1) + 2\operatorname{arctg} x - 2x \quad \text{pro } x \in \mathbb{R}\right)$,
- $\int e^{ax}\sin(bx)\,dx, \quad a,b \in \mathbb{R},\ a,b \neq 0 \quad \left(\overset{c}{=} \frac{e^{ax}\left(a\sin(bx) - b\cos(bx)\right)}{a^2+b^2} \quad \text{pro } x \in \mathbb{R}\right)$.

Příklad 2. Odvoďte rekurentní vzorec pro $I_n = \int \sin^n x\,dx,\ n \in \mathbb{N}$.

$\left(\overset{c}{=} \frac{-1}{n}\left(\sin^{n-1}x\cos x - (n-1)I_{n-2}\right) \quad \text{pro } x \in \mathbb{R}\right)$