Cvičení 12

Příklad 1. Vypočtěte (také jako určité integrály)

- $\displaystyle\int \frac{5x+7}{\sqrt{5-4x-x^2}}\,dx \qquad \left(\overset{c}{=} -5\sqrt{5-4x-x^2} - 3\arcsin\frac{x+2}{3} \quad \text{pro } x\in(-5,1)\right),$
- $\displaystyle\int \frac{2x+1}{\sqrt{-12+7x-x^2}}\,dx \qquad \left(\overset{c}{=} -2\sqrt{-12+7x-x^2} + 8\arcsin(2x-7) \;\; \text{pro } x\in(3,4)\right),$
- $\displaystyle\int \frac{dx}{\sqrt{x}(\sqrt[3]{x}+1)^2}$ (substituce $\sqrt[6]{x}=t$) $\displaystyle\left(\overset{c}{=} 3\,\mathrm{arctg}\,\sqrt[6]{x} - \frac{3\sqrt[6]{x}}{\sqrt[3]{x}+1} \quad \text{pro } x\in(0,\infty)\right),$
- $\displaystyle\int \frac{\sin^3 x}{\cos^4 x}\,dx \qquad \left(\overset{c}{=} \frac{1}{3\cos^3 x} - \frac{1}{\cos x} \quad \text{např. pro } x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right),$
- $\displaystyle\int \frac{1}{\sin^2 x\cos^4 x}\,dx \qquad \left(\overset{c}{=} \frac{1}{3}\mathrm{tg}^3\,x + 2\,\mathrm{tg}\,x - \frac{1}{\mathrm{tg}\,x} \quad \text{např. pro } x\in\left(0,\frac{\pi}{2}\right)\right),$
- $\displaystyle\int \frac{1}{\sin^3 x}\,dx \qquad \left(\overset{c}{=} \frac{1}{2}\ln\left|\mathrm{tg}\,\frac{x}{2}\right| - \frac{\cos x}{2\sin^2 x} \quad \text{např. pro } x\in(0,\pi)\right),$
- $\displaystyle\int \frac{dx}{3^x+1} \qquad \left(\overset{c}{=} x - \frac{\ln(3^x+1)}{\ln 3} \quad \text{pro } x\in\mathbb{R}\right),$
- $\displaystyle\int (2x^3-3x^2-6x+2)e^{2x}\,dx \qquad \left(\overset{c}{=} (x^3-3x^2+1)e^{2x} \quad \text{pro } x\in\mathbb{R}\right),$
- $\displaystyle\int \frac{dx}{x+\sqrt{x^2+x+1}}$ (substituce $\sqrt{x^2+x+1}=x+t$)
  ($\overset{c}{=} \sqrt{x^2+x+1} - x + 2\ln|\sqrt{x^2+x+1}-x-2| - \frac{1}{2}\ln|\sqrt{x^2+x+1}-x-\frac{1}{2}|$
  pro $x\in(-\infty,-1)$ nebo $x\in(-1,\infty)$) ,
- $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}},\ a\neq 0$ (substituce $\sqrt{x^2+a^2}=t-x$)
  $\left(\overset{c}{=} \ln(\sqrt{x^2+a^2}+x) \quad \text{pro } x\in(-\infty,\infty)\right)$.