

PR 3a) cvičení 8.

1

$$g(x,y) = x^4 + y^3 + 2x^2y + 2$$

$$B = [1, -1]$$

Funkce $y(x)$ je definována jako řešení rovnice

$$x^4 + y_{(x)}^3 + 2x^2y_{(x)} + 2 = 0 \quad (1)$$

Najdu výraz pro funkci $y'(x)$ derivací normy podle x

$$4x^3 + 3y^2y' + 4xy + 2x^2y' = 0 \quad (2)$$

$$y' (3y^2 + 2x^2) = \frac{-4x^3 - 4xy}{\boxed{3y^2 + 2x^2}}$$

$$y' = \frac{-4x(x^2 + y)}{3y^2 + 2x^2} = \frac{-4x(x^2 + y)}{3y^2 + 2x^2}$$

Stacionární body jsou tam kde $y' = 0$, tedy

$$\frac{-4x \cdot (x^2 + y)}{3y^2 + 2x^2} = 0 \quad x=0 \text{ a } y=0$$

$$-4x(x^2 + y) = 0$$

$$x=0$$

$$y = -x^2$$

dosaďím obě možnosti do rovnice (1)

$$y^3 + 2 = 0$$

$$x^4 - x^6 + 2x^2(-x^2) + 2 = 0$$

$$y = -\sqrt[3]{2}$$

$$-x^6 - x^4 + 2 = 0$$

1

stacionární body

$$A = [0, -\sqrt[3]{2}], \quad B = [1, -1], \quad C = [-1, -1]$$

$$\begin{aligned}
 & \text{Uhodnu řešení } x_1=1 \quad x_2=-1 \\
 & \text{a dělim kořenovými dělíteli} \\
 & (x-1) \text{ a } (x+1) = x+1 \\
 & \frac{(x^6 + x^4 - 2)}{(x^6 - x^4)} : (x^2 - 1) = \underbrace{x^4 + 2x^2 + 2}_{\text{je vždy,}} \\
 & \quad \text{kladne}
 \end{aligned}$$

PŘ 3a) cvičení 8 list 2.

(2)

Stacionární body dosadím do druhé derivace
 $y''(x)$

Derivujte rovnici (2) podle x ($y(x), y'(x)$ jsou funkce)

$$12x^2 + 6xy' + 3y^2y'' + 4y + 4xy' + 4xy' + 2x^2y'' = 0$$

odtud vypočítám y''

$$y''(3y^2 + 2x^2) = -12x^2 - 6xy' - 4y - 8xy'$$

$$y'' = \frac{-12x^2 - 6xy' - 4y - 8xy'}{3y^2 + 2x^2}$$

Do druhé derivace dosadím postupně body A, B, C
 (mim, že v nich je $y'=0$)

$$A = [0, -\sqrt[3]{2}]$$

$$y' = 0$$

$$y''(0) = \frac{-0 - 0 + 4\sqrt[3]{2} - 0}{3(-\sqrt[3]{2})^2 + 0} = \frac{4}{3\sqrt[3]{2}} > 0$$

v bodě $A = [0, -\sqrt[3]{2}]$ je lokální min.

$$B = [1, -1]$$

$$y' = 0$$

$$y''(1) = \frac{-12 + 4}{3 + 2} = \frac{-8}{5} < 0$$

v bodě $B = [1, -1]$ je lokální max.

$$C = [-1, -1]$$

$$y''(-1) = \frac{-12 + 4}{3 + 2} = \frac{-8}{5} < 0$$

v bodě $C = [-1, -1]$ je lokální max

$$f(x,y) = x^2 + xy + y^2 - 3$$

$\beta = \boxed{1}$

$x = \boxed{0}$

$$y(x) \dots$$

$$x^2 + xy + y^2 - 3 = 0$$

$$1 + 1 + 1 - 3 = 0$$

$$x^2 + xy + y^2 - 3 = 0$$

$$\frac{\partial f}{\partial x} = 2x + 1 \cdot y_{(x)} + x y'_{(x)} + 2y \cdot y'_{(x)} - 3 = 0$$

$$y'_{(x)} = \frac{y_{(x)} - 2x - 0}{x + 2y}$$

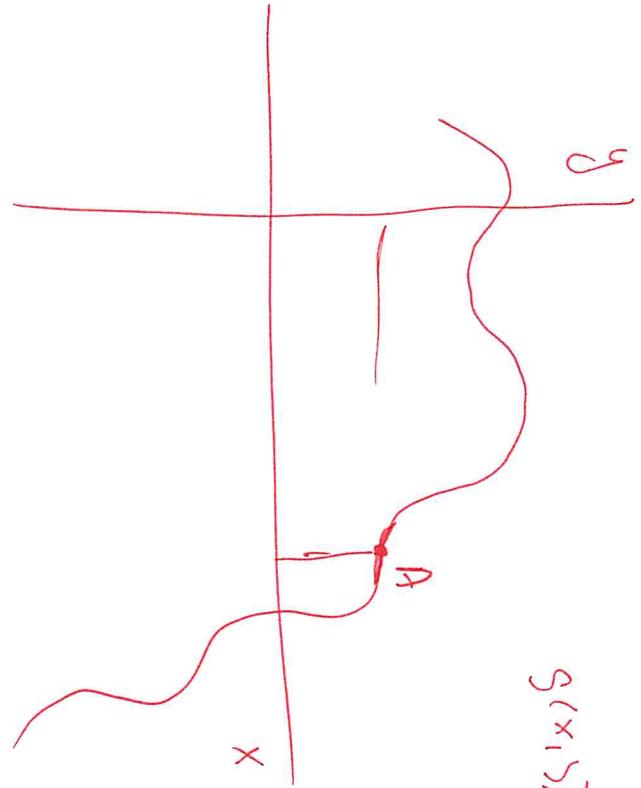
$$y'_{(1)} = \frac{-2 - 1}{1 + 2} = -\frac{3}{3} = -1$$

$$2 + y_1 + y_1 + xy'' + 2y' \cdot y_1 + 2y \cdot y'' = 0$$

$$y''_{(x)}(x+2y) = -2 - 2y' - 2(y')^2$$

$$y''_{(1)} = \frac{-2 - 2(y')^2}{x+2y} = \frac{-2 - 2(-1)^2}{-2 - 2(-1)^2} = \frac{-2 - 2}{-2 + 2} = 0$$

$$\frac{y''(1)}{3} = \frac{-2 + 2 - 2(-1)^2}{1 + 2} = \frac{0}{3} = 0$$



$$0 = (\zeta, \zeta)$$

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PR 2 a)

$$g(x,y,z) = xy - z - e^z + 1$$

$$_0^0 - _0^0 - e^0 + 1 = -1 + 1 = 0$$

$z_{(x,y)}$... implicit' funkce

$$g(x,y,z) = 0$$

$$\frac{\partial}{\partial x} \cdot xy - z_{(x,y)} - e^{z_{(x,y)}} + 1 = 0$$

$$y - \frac{\partial z_{(x,y)}}{\partial x} - e^z \cdot \frac{\partial z}{\partial x} = 0$$

$$y - \frac{\partial z}{\partial x} (1 + e^z) = 0$$

$$\frac{\partial z}{\partial x} (1,0) = \frac{0}{1+e^0} = \frac{0}{2} = 0$$

$$0 = \frac{\partial z}{\partial y} - e^z \cdot \frac{\partial z}{\partial y} = 0$$

$$x - \frac{\partial z}{\partial z} (1+e^z) = 0$$

$$\frac{\partial z}{\partial y} (1,0) = \frac{x}{1+e^0} = \frac{x}{2}$$

$$\frac{\partial z}{\partial x} (1,0) = \frac{1}{1+e^0} = \frac{1}{2}$$

$$\mathcal{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

6.

PP 3 a)

$$B = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$g(x,y) = x^4 + y^3 + 2x^2y + 2$$

$$y'(x) = \dots \text{ implicit form}$$

$$x^4 + y_{(x)}^3 + 2x^2y_{(x)} + 2 = 0$$

$$\frac{\partial}{\partial x}$$

$$4x^3 + 3y^2 \cdot y' + 4xy + 2x^2y' = 0$$

$$y' = \frac{-4x^3 - 4xy}{3y^2 + 2x^2} = 0$$

$$\left. \begin{array}{l} -4x^3 - 4xy = 0 \\ 3y^2 + 2x^2 = 0 \end{array} \right\}$$

Chyb.

$$-4x^3 - 4xy = 0$$

$$4x(x^2 + y) = 0$$

move ① ②

$$y = x^2$$

$$y = \beta$$

$$x^4 + y^3 + 2x^2y + 2 = 0$$

$$y^3 + 2 = 0$$

$$x^4 + x^6 + 4x^4 + 2 = 0$$

$$x^6 + 5x^4 + 2 = 0$$

$$t = x^2$$

$$[0, -\frac{3}{2}]$$

min

$$0 = \|y\| = \frac{12x^2 + 6yy'y' + 3y^2y'' + 4y + 4xy'y' + 4xy + 2x^2y' + 2x^2y''}{3y^2 + 2x^2} > 0$$

7.

PRÜF a)

$$g(x,y) = xy + \ln y - 1$$

$$B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$g'(x,y) = 1 + \ln 1 - 1 = 1 - 1 = 0$$

$$x y_1 + \ln y_1 - 1 = 0$$

$$y'_1$$

$$1 \cdot y + x y'_1 + \frac{1}{y} x + y \cdot 1 = 0$$

$$y'_1(x+1) = -y$$

$$y'_1 = \frac{-y}{x+1}$$

techn.

$$y = -\frac{1}{2}x + q$$

BET

$$1 = -\frac{1}{2} \cdot 1 + q$$

$$q = \frac{3}{2}$$

techn.

$$y = -\frac{1}{2}x + \frac{3}{2}$$

$$1 = 2 + q$$

$$y = 2x + q$$

\Rightarrow

$$K_n = \frac{\frac{2}{1}}{\frac{1}{2} + \frac{1}{1}} = 2$$

$$\overbrace{y = 2x + 1}^{y = 2x + q} = y$$

$$1 = 2 + q$$

$$\overbrace{y = 2x + 1}^{y = 2x + q} = y$$

$$y'_1(1) = \frac{-1}{1+1} = -\frac{1}{2} = k$$

normal

fl

techn.

$$y = -\frac{1}{2}x + q$$

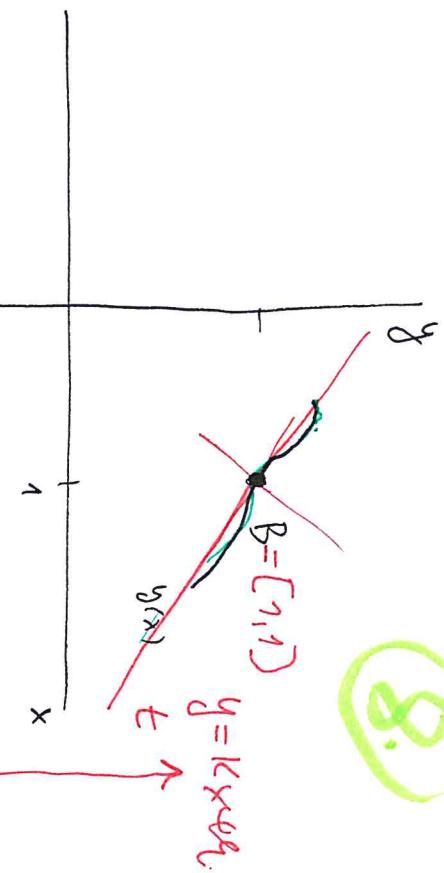
BET

$$1 = -\frac{1}{2} \cdot 1 + q$$

$$q = \frac{3}{2}$$

techn.

$$y = -\frac{1}{2}x + \frac{3}{2}$$



8.

$$\bar{P} \in \mathcal{A}$$

$$A = [a_1, a_2]$$

$$z - f(A) = \frac{\partial f(A)}{\partial x} \cdot (x - a_1) + \frac{\partial f(A)}{\partial y} \cdot (y - a_2)$$

$$f(x, y, z) = x^2 - y^2 + z^2 - 6$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

Q.

$$B = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$f(a_1, a_2)$$

$z(x, y) \dots$ impliziert

$$x^2 - y^2 + z^2 - 6 = 0$$

$$\frac{\partial z}{\partial x}$$

$$2x - 2z \frac{\partial z}{\partial x} - 2x = 0$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{2x}{2z}$$

$$(t)$$

$$x^2 - y^2 + z^2 - 6 = 0$$

$$0 = \frac{\partial z}{\partial y} + 2z \frac{\partial z}{\partial x} - 2y$$

$$x^2 - y^2 + z^2 - 6 = 0$$

$$\frac{\partial z}{\partial y}$$

$$z - (-3) = -\frac{1}{3}(x-1) - \frac{2}{3}(y-2)$$

$$\frac{z-3}{3} = -\frac{x}{3} + \frac{1}{3} - \frac{2y}{3} + \frac{4}{3}$$

$$3z + 9 = -x + 1 - 2y + 4$$

$$0 = y + 2y + 3z + 4$$

PP 6 a)

$$f(x,y) = 1 + 6y - y^2 - xy - x^2$$

$$\frac{\partial f}{\partial x}(x,y) = -y + 2x = 0$$

$$y = -2x$$

$$\frac{\partial f}{\partial y}(x,y) = 6 - 2y - x = 0$$

$$6 - 2(-2x) - x = 0$$

$$6 + 4x - x = 0$$

$$6 + 3x = 0$$

$$x = -2$$

$$y = -2(-2) = 4$$

$$D_1(x,y) = -2$$

$$D_2(x,y) = \begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix} = 4 - 1 = 3$$

stic. bod $A = [-2, 4]$
je Punkt max

10.

$$A = \begin{bmatrix} v & v & v \\ 1 & -1 & 1 \end{bmatrix}$$

stationary and

$$\overline{v} = 2$$

$$v = 2 + z = -1$$

$$2y + z = -1$$

$$\begin{array}{l} \overline{v} = - \\ \overline{y} = 1 \\ \overline{z} = -3 \end{array}$$

$$\left. \begin{array}{l} v = z + p \\ y = -z - p \end{array} \right\} \quad \text{(2)}$$

$$\left. \begin{array}{l} v = z + p \\ y = z - p \end{array} \right\} \quad \text{(1)}$$

$$\left. \begin{array}{l} v = -2 \\ y = -1 \end{array} \right\} \quad \text{(2-1)}$$

$$\begin{array}{l} \overline{x} = 1 \\ \overline{z} = 2 \end{array}$$

$$f(x_1, y_1, z_1) = x_1^2 + y_1^2 + z_1^2 + xy - 2x - 2y$$

$$\begin{aligned} 0 &= v - p + z = \frac{\partial f}{\partial v}(x_1, y_1, z_1) = 2x_1 + z_1 + 1 = 0 \\ 0 &= v + z + p_2 = \frac{\partial f}{\partial v}(x_1, y_1, z_1) = 2y_1 + z_1 + 1 = 0 \\ 0 &= 2 - x_2 = \frac{\partial f}{\partial v}(x_1, y_1, z_1) = x_1^2 + y_1^2 + z_1^2 = 0 \end{aligned}$$

$$D\bar{f} = a$$

1.

2.

$$D_1 = 2$$

$$\frac{\partial^2 f}{\partial x^2} (x_1, y_1, z) = 2$$

$$D_2 = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$D_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 2(4 - 1) = 6$$

$$\frac{\partial^2 f}{\partial z^2} (x_1, y_1, z) = 0$$

$$A = \begin{bmatrix} 1, -1, 1 \end{bmatrix} \dots \text{ folgt min. minimum}$$

$$\frac{\partial^2 f}{\partial x^2} (x_1, y_1, z) = 1$$

$$\frac{\partial^2 f}{\partial y^2} (x_1, y_1, z) = 1$$