Matematika II (KMD/MA2) - cvičení 11

FAKULTA STROJNÍ (akad. rok 2019/2020 a vyšší)

Příklad 1. Nalezněte obecné řešení lineární diferenciální rovnice 1. řádu a řešení Cauchyho úlohy:

a) $y' + 3y = x,\ y(0) = -1/9,$ $\left[y = Ce^{-3x} + \frac{x}{3} - \frac{1}{9},\ C \in \mathbb{R};\ C = 0\right]$

b) $y' + 2xy = xe^{-x^2},\ y(0) = 1,$ $\left[y = e^{-x^2}\left(C + \frac{x^2}{2}\right),\ C \in \mathbb{R};\ C = 1\right]$

c) $y' + x^2y = x^2,\ y(0) = -1,$ $\left[y = 1 + Ce^{-\frac{x^3}{3}},\ C \in \mathbb{R};\ C = -1\right]$

d) $y' + y = \cos x,\ y(0) = 1,$ $\left[y = Ce^{-x} + \frac{1}{2}(\sin x + \cos x),\ C \in \mathbb{R};\ C = \frac{1}{2}\right]$

e) $y' - \frac{2}{x}y = 2x^3,\ y(1) = 0,$ $\left[y = x^4 + Cx^2,\ C \in \mathbb{R};\ C = -1\right]$

f) $y' + 3y = e^{2x},\ y(0) = 1,$ $\left[y = Ce^{-3x} + \frac{e^{2x}}{5},\ C \in \mathbb{R};\ C = \frac{4}{5}\right]$

g) $y' - \frac{2x}{x^2+1}y = x^2 + 1,\ y(1) = 1,$ $\left[y = (x + C)(x^2 + 1),\ C \in \mathbb{R};\ C = -\frac{1}{2}\right]$

h) $xy' = y + x^2,\ y(1) = 0,$ $\left[y = x^2 + Cx,\ C \in \mathbb{R};\ C = -1\right]$

i) $y' + 2y\operatorname{tg} x = 2\sin x,\ y(\pi) = 1,$ $\left[y = C\cos^2 x + 2\cos x,\ C \in \mathbb{R};\ C = 3\right]$

j) $xy' + y = \ln x + 1,\ y(1) = 0,$ $\left[y = \ln x + \frac{C}{x},\ C \in \mathbb{R};\ C = 0\right]$

k) $y' - y\operatorname{tg} x = 1 - x\operatorname{tg} x,\ y(0) = 1,$ $\left[y = \frac{C}{\cos x} + x,\ C \in \mathbb{R};\ C = 1\right]$

l) $xy' + y = \operatorname{arctg} x + \frac{x}{1+x^2},\ y(1) = \pi/4,$ $\left[y = \frac{C}{x} + \operatorname{arctg} x,\ C \in \mathbb{R};\ C = 0\right]$

m) $y' + \frac{x+1}{x}y = 3xe^{-x},\ y(1) = 3/e,$ $\left[y = x^2e^{-x} + \frac{C}{x}e^{-x},\ C \in \mathbb{R};\ C = 2\right]$

n) $y' - y = xe^x\cos x,\ y(0) = 1,$ $\left[y = Ce^x + e^x\cos x + xe^x\sin x,\ C \in \mathbb{R};\ C = 0\right]$

o) $(2x + 1)y' + y = x,\ y(0) = 1,$ $\left[y = \frac{1}{3}(x - 1) + \frac{C}{\sqrt{|2x+1|}},\ C \in \mathbb{R};\ C = \frac{4}{3}\right]$