Matematika II (KMD/MA2) - cvičení 5

FAKULTA STROJNÍ (akad. rok 2019/2020 a vyšší)

Příklad 1. Najděte vlastní čísla a vlastní vektory matice $\mathbb{A}$.

a) $\mathbb{A} = \begin{pmatrix} 0 & -3 & -3 \\ 2 & 5 & -3 \\ 0 & 0 & 1 \end{pmatrix}$ $\left[\lambda_1 = 1, \mathbf{v}_1 = \begin{pmatrix} 21 \\ -9 \\ 2 \end{pmatrix}, \quad \lambda_2 = 2, \mathbf{v}_2 = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}, \quad \lambda_3 = 3, \mathbf{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right]$

b) $\mathbb{A} = \begin{pmatrix} 1 & 0 & 0 \\ -5 & -4 & -1 \\ 4 & 4 & 1 \end{pmatrix}$ $\left[\lambda_1 = -3, \mathbf{v}_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad \lambda_2 = 0, \mathbf{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}, \quad \lambda_3 = 1, \mathbf{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right]$

c) $\mathbb{A} = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 1 & 0 \\ 5 & 4 & 3 \end{pmatrix}$

$$\left[\lambda_1 = 7, \mathbf{v}_1 = \begin{pmatrix} 12 \\ 2 \\ 17 \end{pmatrix}, \quad \lambda_2 = -1 - \sqrt{2}, \mathbf{v}_2 = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} - 1 \\ 1 \end{pmatrix}, \quad \lambda_3 = -1 + \sqrt{2}, \mathbf{v}_3 = \begin{pmatrix} \sqrt{2} \\ -1 - \sqrt{2} \\ 1 \end{pmatrix}\right]$$

d) $\mathbb{A} = \begin{pmatrix} 0 & -1 & 5 \\ 0 & -1 & 4 \\ 2 & -5 & 5 \end{pmatrix}$ $\left[\lambda_{1,2} = 1, \mathbf{v}_1 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \quad \lambda_3 = 2, \mathbf{v}_2 = \begin{pmatrix} 11 \\ 8 \\ 6 \end{pmatrix}\right]$

e) $\mathbb{A} = \begin{pmatrix} 1 & 2 & 2 & -1 \\ 2 & 1 & -2 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 5 & 4 \end{pmatrix}$ $\left[\lambda_{1,2} = -1, \mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \lambda_3 = 3, \mathbf{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \lambda_4 = 7, \mathbf{v}_4 = \begin{pmatrix} 1 \\ -1 \\ 24 \\ 40 \end{pmatrix}\right]$

f) $\mathbb{A} = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & -2 \\ 4 & 2 & 0 \end{pmatrix}$ $\left[\lambda_1 = 0, \mathbf{v}_1 = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}, \quad \lambda_2 = 2i, \mathbf{v}_2 = \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix}, \quad \lambda_3 = -2i, \mathbf{v}_3 = \begin{pmatrix} 0 \\ -i \\ 1 \end{pmatrix}\right]$