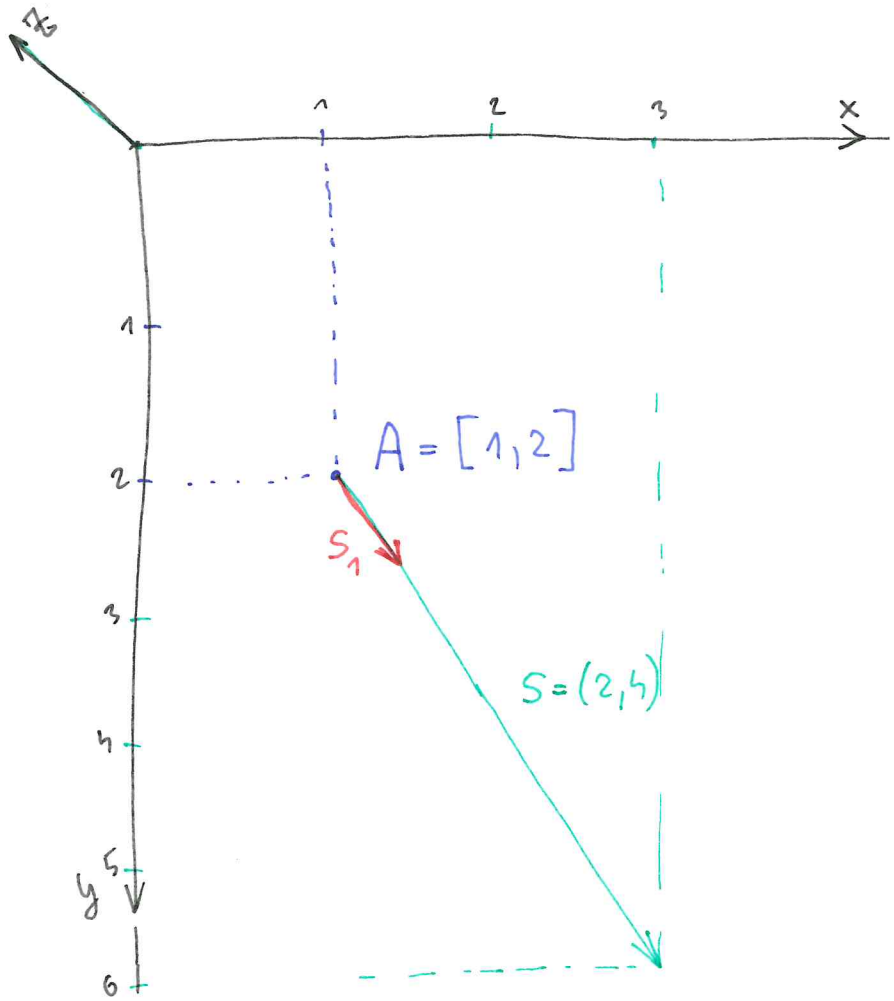

Určete směrovou derivaci funkce

$f(x,y) = x^2 + xy + 2y^2 - 1$

v bodě $A=[1,2]$ ve směru vektoru $s=(2,4)$.

$$|s| = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$$

$$s_1 = \frac{1}{\sqrt{20}}(2,4) = \left(\frac{2}{\sqrt{20}} ; \frac{4}{\sqrt{20}}\right) = \left(\frac{1}{\sqrt{5}} ; \frac{2}{\sqrt{5}}\right)$$

$$x = 1 + \frac{1}{\sqrt{5}}t$$

$$y = 2 + \frac{2}{\sqrt{5}}t$$

$$g_s(t) = f\left(1+\frac{1}{\sqrt{5}}t,\ 2+\frac{2}{\sqrt{5}}t\right) =$$

$$= \left(1+\frac{1}{\sqrt{5}}t\right)^2 + \left(1+\frac{1}{\sqrt{5}}t\right)\cdot\left(2+\frac{2}{\sqrt{5}}t\right) + 2\left(2+\frac{2}{\sqrt{5}}t\right)^2 - 1 =$$

$$= \left(1+\frac{1}{\sqrt{5}}t\right)^2 + 2\left(1+\frac{1}{\sqrt{5}}t\right)^2 + 8\left(1+\frac{1}{\sqrt{5}}t\right)^2 - 1 =$$

$$= 11\left(1+\frac{1}{\sqrt{5}}t\right)^2 - 1 = 11 + \frac{22}{\sqrt{5}}t + \frac{11}{5}t^2 - 1 =$$

$$= \frac{11}{5}t^2 + \frac{22}{\sqrt{5}}t + 10$$

$$g_s'(t) = \frac{22}{5}t + \frac{22}{\sqrt{5}}$$

směrová derivace funkce F v bodě A a směru s

$$\frac{\partial f}{\partial s}(A) = g_s'(0) = \frac{22}{\sqrt{5}}$$

směrnice tečny v bodě A a směru s

$$k_t = \frac{\partial f}{\partial s}(A) = \frac{22}{\sqrt{5}} = 9{,}83$$

úhel tečny od roviny xy

$$\varphi = \operatorname{arctg} 9{,}83 \doteq 84{,}2^\circ$$