Úredn 8 1.

$$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$$

$$\frac{\partial f}{\partial x} = 6x^2 + y^2 + 10x = 0$$

$$\frac{\partial f}{\partial y} = 2xy + 2y = 0$$

$$y(x+1) = 0$$

$y = 0$ $\qquad$ $x = -1$

$6x^2 + 10x = 0$ $\qquad$ $6 + y^2 - 10 = 0$

$x(3x+5) = 0$ $\qquad$ $y^2 = 4$

$x_1 = 0$ $\qquad$ $x_2 = -\frac{5}{3}$ $\qquad$ $y_1 = 2$ $\qquad$ $y_2 = -2$

$A = [0,0]$ $\qquad$ $B = [-\frac{5}{3}, 0]$ $\qquad$ $C = [-1, 2]$ $\qquad$ $D = [-1, -2]$

$$\frac{\partial^2 f}{\partial x^2} = 12x + 10 \qquad D_1 = 6x + 10$$

$$\frac{\partial^2 f}{\partial y^2} = 2x + 2 \qquad D_2 = \begin{vmatrix} 6x+10 & 2y \\ 2y & 2x+2 \end{vmatrix}$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2y$$

$A = [0,0]$ $\qquad$ $D_1(A) = 10$ $\qquad$ $D_2(A) = \begin{vmatrix} 10 & 0 \\ 0 & 2 \end{vmatrix} = 20$ $\qquad$ lokální minimum

$B = [-\frac{5}{3}, 0]$ $\qquad$ $D_1(B) = -10$ $\qquad$ $D_2(B) = \begin{vmatrix} -10 & 0 \\ 0 & -\frac{4}{3} \end{vmatrix} = \frac{40}{3}$ $\qquad$ lokální maximum

$C = [-1, 2]$ $\qquad$ $D_1(C) = 2$ $\qquad$ $D_2(C) = \begin{vmatrix} -2 & 4 \\ 4 & 0 \end{vmatrix} = -16$ $\qquad$ nic

$D_4 = [-1, -2]$ $\qquad$ $D_1(D) = -2$ $\qquad$ $D_2(D) = \begin{vmatrix} -2 & -4 \\ -4 & 0 \end{vmatrix} = -16$ $\qquad$ nic

$$f(x,y) = x^2 - y^2$$

vazba

$$2x - y + 1 = 0$$

$$y = 2x + 1$$

$$h(x) = x^2 - (2x+1)^2 = x^2 - 4x^2 - 4x - 1 = -3x^2 - 4x - 1$$

$$h'(x) = -6x - 4 = 0$$

$$x = -\frac{4}{6} = -\frac{2}{3}$$

$h''(x) = -6$ tj. $x = -\frac{2}{3}$ je lok. max $h(x)$

$$y = 2x + 1 = -\frac{4}{3} + 1 = -\frac{1}{3}$$

$A = \left[-\frac{2}{3}, -\frac{1}{3}\right]$ vázané lok. max funkce $f(x,y)$ s vazbou $2x - y + 1 = 0$

Předn. 8 3.

$$F(x,y,\lambda) = x^2 + 2y^2 + \lambda(x^2 - 2x + 2y^2 - 4y)$$

$$\frac{\partial F}{\partial x} = 2x + 2\lambda(x-1) = 0 \quad \Rightarrow \quad x = \frac{2\lambda}{2+2\lambda} = \frac{\lambda}{1+\lambda}$$

$$\frac{\partial F}{\partial y} = 4y + 4\lambda(y-1) = 0 \quad \Rightarrow \quad y = \frac{4\lambda}{4+4\lambda} = \frac{\lambda}{1+\lambda}$$

$$\frac{\partial F}{\partial \lambda} = x^2 - 2x + 2y^2 - 4y = 0$$

dosad do (3)

$$\frac{\lambda^2}{(1+\lambda)^2} - \frac{2\lambda}{1+\lambda} + \frac{2\lambda^2}{(1+\lambda)^2} - \frac{4\lambda}{1+\lambda} = 0 \qquad / \cdot (1+\lambda)^2$$

$$\lambda^2 - 2\lambda(1+\lambda) + 2\lambda^2 - 4\lambda(1+\lambda) = 0$$

$$3\lambda^2 - 2\lambda - 2\lambda^2 - 4\lambda - 4\lambda^2 = 0$$

$$-3\lambda^2 - 6\lambda = 0$$

$$\lambda \cdot (\lambda + 2) = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = -2$$

$\lambda_1 = 0$ $\quad x = y = 0$ $\quad A = [0,0]$

$\lambda_2 = -2$ $\quad x = y = \frac{-2}{1-2} = 2$ $\quad B = [2,2]$

$\Big\}$ stacionární body

$$\frac{\partial^2 F}{\partial x^2} = 2 + 2\lambda$$

$$\frac{\partial^2 F}{\partial y^2} = 4 + 4\lambda$$

$$\frac{\partial^2 F}{\partial x \partial y} = 0$$

$D_1(A) = 2$ $\qquad D_2(A) = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8$

v bodě A je vázané lok. minimum

$D_1(B) = -2$ $\qquad D_2(B) = \begin{vmatrix} -2 & 0 \\ 0 & -4 \end{vmatrix} = 8$

v bodě B je vázané lok. maximum

s vazbou $x^2 - 2x + 2y^2 - 4y = 0$